

CHAPTER 12 FORM C

Name _____ Course Number: _____ Section Number: _____

Directions: Answer the questions and solve the problems in the spaces provided, or attach paper. Circle the correct choice for each response set. Where appropriate, use computer assistance.

Provide an appropriate response.

1) List the assumptions for testing hypotheses that three or more means are equivalent.

2) The test statistic for one-way ANOVA is $F = \frac{\text{variance between samples}}{\text{variance within samples}}$. Describe variance within samples and variance between samples. What relationship between variance within samples and variance between samples would result in the conclusion that the value of F is significant?

3) What is a Bonferroni test? Why do we use it?

4) Why is it unnecessary to conduct multiple comparison tests after a nonsignificant F test statistic results?

Given below are the analysis of variance results from a Minitab display. Assume that you want to use a 0.05 significance level in testing the null hypothesis that the different samples come from populations with the same mean.

5) Identify the p-value.

| Source | DF | SS | MS | F | p |
|--------|----|--------|-------|------|-------|
| Factor | 3 | 13.500 | 4.500 | 5.17 | 0.011 |
| Error | 16 | 13.925 | 0.870 | | |
| Total | 19 | 27.425 | | | |

- A) 5.17 B) 0.870 C) 0.011 D) 4.500

6) Identify the value of the test statistic.

| Source | DF | SS | MS | F | p |
|--------|----|----|-------|-----|-------|
| Factor | 3 | 30 | 10.00 | 1.6 | 0.264 |
| Error | 8 | 50 | 6.25 | | |
| Total | 11 | 80 | | | |

- A) 1.6 B) 0.264 C) 30 D) 10.00

Test the claim that the samples come from populations with the same mean. Assume that the populations are normally distributed with the same variance.

7) Given the sample data below, test the claim that the populations have the same mean. Use a significance level of 0.05.

| Brand A | Brand B | Brand C | Brand D |
|------------------|------------------|------------------|------------------|
| $n = 16$ | $n = 16$ | $n = 16$ | $n = 16$ |
| $\bar{x} = 2.09$ | $\bar{x} = 3.48$ | $\bar{x} = 1.86$ | $\bar{x} = 2.84$ |
| $s = 0.37$ | $s = 0.61$ | $s = 0.45$ | $s = 0.53$ |

- 8) At the 0.025 significance level, test the claim that the four brands have the same mean if the following sample results have been obtained.

| <u>Brand A</u> | <u>Brand B</u> | <u>Brand C</u> | <u>Brand D</u> |
|----------------|----------------|----------------|----------------|
| 15 | 20 | 21 | 15 |
| 25 | 17 | 22 | 15 |
| 21 | 22 | 20 | 14 |
| 23 | 23 | 19 | 23 |
| 22 | | 18 | 22 |
| 20 | | | 28 |
| | | | 28 |

Provide an appropriate response.

9) Fill in the missing entries in the following partially completed one-way ANOVA table.

| Source | df | SS | MS=SS/df | F-statistic |
|-----------|----|------|----------|-------------|
| Treatment | | 26.3 | | |
| Error | 25 | | 4 | |
| Total | 29 | | | |

A)

| Source | df | SS | MS=SS/df | F-statistic |
|-----------|----|-------|----------|-------------|
| Treatment | 4 | 26.3 | 6.58 | 0.61 |
| Error | 25 | 100.0 | 4 | |
| Total | 29 | 126.3 | | |

B)

| Source | df | SS | MS=SS/df | F-statistic |
|-----------|----|-------|----------|-------------|
| Treatment | 4 | 26.3 | 6.58 | 1.65 |
| Error | 25 | 100.0 | 4 | |
| Total | 29 | 126.3 | | |

C) 1.65

| Source | df | SS | MS=SS/df | F-statistic |
|-----------|----|-------|----------|-------------|
| Treatment | 4 | 26.3 | 6.58 | 1.65 |
| Error | 25 | 100.0 | 4 | |
| Total | 29 | 26.46 | | |

D)

| Source | df | SS | MS=SS/df | F-statistic |
|-----------|----|-------|----------|-------------|
| Treatment | 54 | 26.3 | 0.49 | 355.05 |
| Error | 25 | 100.0 | 4 | |
| Total | 29 | 126.3 | | |

Use the Minitab display to test the indicated claim.

- 10) A manager records the production output of three employees who each work on three different machines for three different days. The sample results are given below and the Minitab results follow.

| | | Employee | | |
|---------|-----|------------|------------|------------|
| | | A | B | C |
| Machine | I | 23, 27, 29 | 30, 27, 25 | 18, 20, 22 |
| | II | 25, 26, 24 | 24, 29, 26 | 19, 16, 14 |
| | III | 28, 25, 26 | 25, 27, 23 | 15, 11, 17 |

ANALYSIS OF VARIANCE ITEMS

| SOURCE | DF | SS | MS |
|-------------|----|--------|--------|
| MACHINE | 2 | 34.67 | 17.33 |
| EMPLOYEE | 2 | 504.67 | 252.33 |
| INTERACTION | 4 | 26.67 | 6.67 |
| ERROR | 18 | 98.00 | 5.44 |
| TOTAL | 26 | 664.00 | |

Using a 0.05 significance level, test the claim that the interaction between employee and machine has no effect on the number of items produced.

- 11) A manager records the production output of three employees who each work on three different machines for three different days. The sample results are given below and the Minitab results follow.

| | | Employee | | |
|---------|-----|------------|------------|------------|
| | | A | B | C |
| Machine | I | 16, 18, 19 | 15, 17, 20 | 14, 18, 16 |
| | II | 20, 27, 29 | 25, 28, 27 | 29, 28, 26 |
| | III | 15, 18, 17 | 16, 16, 19 | 13, 17, 16 |

ANALYSIS OF VARIANCE ITEMS

| SOURCE | DF | SS | MS |
|-------------|----|--------|--------|
| MACHINE | 2 | 588.74 | 294.37 |
| EMPLOYEE | 2 | 2.07 | 1.04 |
| INTERACTION | 4 | 15.48 | 3.87 |
| ERROR | 18 | 98.67 | 5.48 |
| TOTAL | 26 | 704.96 | |

Assume that the number of items produced is not affected by an interaction between employee and machine. Using a 0.05 significance level, test the claim that the choice of employee has no effect on the number of items produced.

- 12) A manager records the production output of three employees who each work on three different machines for three different days. The sample results are given below and the Minitab results follow.

| | | Employee | | |
|---------|-----|------------|------------|------------|
| | | A | B | C |
| Machine | I | 31, 34, 32 | 29, 23, 22 | 21, 20, 24 |
| | II | 19, 26, 22 | 35, 33, 30 | 25, 19, 23 |
| | III | 21, 18, 26 | 20, 23, 24 | 36, 37, 31 |

ANALYSIS OF VARIANCE ITEMS

| SOURCE | DF | SS | MS |
|-------------|----|--------|--------|
| MACHINE | 2 | 1.19 | .59 |
| EMPLOYEE | 2 | 5.85 | 2.93 |
| INTERACTION | 4 | 710.81 | 177.70 |
| ERROR | 18 | 160.00 | 8.89 |
| TOTAL | 26 | 877.85 | |

Assume that the number of items produced is not affected by an interaction between employee and machine. Using a 0.05 significance level, test the claim that the machine has no effect on the number of items produced.

Use the data in the given table and the corresponding Minitab display to test the hypothesis.

- 13) The following table shows the mileage for four different cars and three different brands of gas. Assuming no effect from the interaction between car and brand of gas, test the claim that the four cars have the same mean mileage. Use a 0.05 significance level.

| | Brand 1 | Brand 2 | Brand 3 |
|-------|---------|---------|---------|
| Car 1 | 22.4 | 25.2 | 24.3 |
| Car 2 | 19 | 18.6 | 19.8 |
| Car 3 | 24.6 | 25 | 25.4 |
| Car 4 | 23.5 | 23.6 | 24.1 |

| Source | DF | SS | MS | F | p |
|--------|----|--------|--------|--------|----------|
| Car | 3 | 61.249 | 20.416 | 39.033 | 0.000249 |
| Gas | 2 | 2.222 | 1.111 | 2.124 | 0.200726 |
| Error | 6 | 3.138 | 0.523 | | |
| Total | 11 | 66.609 | | | |

- 14) The following Minitab display results from a study in which three different teachers taught calculus classes of five different sizes. The class average was recorded for each class. Assuming no effect from the interaction between teacher and class size, test the claim that the teacher has no effect on the class average. Use a 0.05 significance level.

| Source | DF | SS | MS | F | p |
|------------|----|--------|--------|-------|-------|
| Teacher | 2 | 56.93 | 28.47 | 1.018 | 0.404 |
| Class Size | 4 | 672.67 | 168.17 | 6.013 | 0.016 |
| Error | 8 | 223.73 | 27.97 | | |
| Total | 14 | 953.33 | | | |

Provide an appropriate response.

- 15) The following data shows annual income, in thousands of dollars, categorized according to the two factors of gender and level of education. Assume that incomes are not affected by an interaction between gender and level of education, and test the null hypothesis that gender has no effect on income. Use a 0.05 significance level.

| | Female | Male |
|-----------------|----------------|----------------|
| High school | 23, 27, 24, 26 | 25, 26, 22, 24 |
| College | 28, 36, 31, 33 | 35, 32, 39, 28 |
| Advanced degree | 41, 38, 43, 49 | 35, 50, 47, 44 |

- 16) The following data shows the yield, in bushels per acre, categorized according to three varieties of corn and three different soil conditions. Test the null hypothesis of no interaction between variety and soil conditions at a significance level of 0.05.

| | Plot 1 | Plot 2 | Plot 3 |
|-----------|-----------------------|-----------------------|-----------------------|
| Variety 1 | 156, 167, 170, 162 | 162, 160, 169, 168 | 145, 151, 148, 155 |
| Variety 2 | 172, 176, 166, 179 | 179, 186, 160, 176 | 161, 162, 165, 170 |
| Variety 3 | 175, 157, 179, 178 | 178, 170, 172, 174 | 169, 165, 170, 169 |

- 17) The following data contains task completion times, in minutes, categorized according to the gender of the machine operator and the machine used.

| | Male | Female |
|-----------|--------|--------|
| Machine 1 | 15, 17 | 16, 17 |
| Machine 2 | 14, 13 | 15, 13 |
| Machine 3 | 16, 18 | 17, 19 |

The ANOVA results lead us to conclude that the completion times are not affected by an interaction between machine and gender, and the times are not affected by gender, but they are affected by the machine. Change the table entries so that there is no effect from the interaction between machine and gender, but there is an effect from the gender of the operator.

- 18) The following data contains task completion times, in minutes, categorized according to the gender of the machine operator and the machine used.

| | Male | Female |
|-----------|--------|--------|
| Machine 1 | 15, 17 | 16, 17 |
| Machine 2 | 14, 13 | 15, 13 |
| Machine 3 | 16, 18 | 17, 19 |

Assume that two-way ANOVA is used to analyze the data. How are the ANOVA results affected if 5 minutes is subtracted from each completion time?

19) **Provide an appropriate response.**

What advantages does two-way ANOVA have over one-way ANOVA?

20) **Provide an appropriate response.**

How can researchers control for the unintended effects of extraneous factors interfering with their experimental design in one-way ANOVA?

Answer Key

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- 1) 1) The populations have normal distributions.
 - 2) The populations have the same variance σ^2 (or standard deviation σ).
 - 3) The samples are random and independent of each other.
 - 4) The different samples are from populations that are categorized in only one way. (The requirements of normality and equal variances are somewhat relaxed.)
- 2) Variance between samples measures the variation between the sample means, that is the variation due to the treatment. The variance within the samples depends solely on the sample variances and is a measure of pooled variation. The F ratio compares the two. If the F ratio is relatively close to 1, the two variances are about the same, and we conclude that there are no significant differences among the sample means. When the value of F is excessively large (that is, greater than 1), we conclude that the variation among the samples is not the same and that the means are not equal.
- 3) The Bonferroni tests is one of several tests of multiple comparisons designed to determine which population mean or means is different from the others. This test is used after a significant F ratio results from an ANOVA procedure.
- 4) A nonsignificant F ratio suggests that the population means are equal. Consequently, there is no need to determine which mean or means are different from the others.
- 5) C
- 6) A
- 7) Test statistic: $F = 35.1573$. Critical value: $F = 2.7581$.
Reject the claim of equal means. The different brands do not appear to have the same mean.
- 8) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$. H_1 : The means are not all equal. P-value: $p = 0.982$.
Test statistic: $F = 0.0555$. Critical value: $F = 3.9539$.
Fail to reject the null hypothesis. There is not sufficient evidence to warrant rejection of the claim that the four brands have the same mean.
- 9) B
- 10) H_0 : There is no interaction effect.
 H_1 : There is an interaction effect.
Test statistic: $F = 1.2261$. Critical value: $F = 2.9277$.
Fail to reject the null hypothesis. There does not appear to be an interaction effect.
- 11) H_0 : There is no employee effect.
 H_1 : There is an employee effect.
Test statistic: $F = 0.1898$. Critical value: $F = 3.5546$.
Fail to reject the null hypothesis. There does not appear to be an employee effect.
- 12) H_0 : There is no machine effect.
 H_1 : There is a machine effect.
Test statistic: $F = 0.0664$. Critical value: $F = 3.5546$.
Fail to reject the null hypothesis. The type of machine does not appear to have an effect on the number of items produced.
- 13) H_0 : The cars have the same mean mileage. H_1 : The cars do not have the same mean mileage.
The P-value is 0.000249, which is less than 0.05. We reject the null hypothesis; it appears that the cars do not have the same mileage.
- 14) H_0 : There is no teacher effect. H_1 : There is a teacher effect. The P-value is 0.404, which is greater than 0.05. We fail to reject the null hypothesis; it appears that the teacher does not affect the class average.
- 15) H_0 : Gender has no effect on income. H_1 : Gender has an effect on income. The test statistic is $F = 0.1556$, and the corresponding P-value is 0.6979. Because the P-value is greater than 0.05, we fail to reject the null hypothesis that gender has no effect on income.

Answer Key

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- 16) H_0 : There is no interaction between variety and soil conditions. H_1 : There is an interaction between variety and soil conditions. The test statistic is $F = 0.803973$, and the corresponding P-value is 0.533402. Because the P-value is greater than 0.05, we fail to reject the null hypothesis of no interaction between variety and soil conditions.
- 17) The following table is one example of entries that produce an effect from the gender of the operator.

| | Male | Female |
|-----------|--------|--------|
| Machine 1 | 15, 17 | 14, 14 |
| Machine 2 | 14, 18 | 10, 10 |
| Machine 3 | 16, 16 | 12, 12 |

- 18) The ANOVA results are not affected by subtracting 5 minutes from each completion time. The null hypothesis of no interaction between machine and gender is not rejected since the p-value is 0.946. The null hypothesis of no effect from machine is rejected since the P-value is 0.013. The null hypothesis of no effect from gender is not rejected since the P-value is 0.382.
- 19) By adding a second factor to the analysis, researchers can see whether there is an interaction between the two factors and whether one or the other or both have significant main effects. For example, in a two-way analysis of age and gender with regard to a new pill for lowering high blood pressure, it would be helpful to know that only senior men have received significant benefits. So, two-way ANOVA offers exploration of two variables rather than one.
- 20) Since researchers want to know about the effects of the factor under investigation, they can use a *completely randomized design*, where participants are randomly selected and randomly assigned to the groups of the factor. Randomization tends to spread characteristics, unwanted to the research, among the groups and thereby diminishes influence of extraneous factors. Another method is the *rigorously controlled design*, in which researchers look ahead to control all factors, whose variability might interfere with the results of the experiment. Such control could be effected statistically.