

Given below are the analysis of variance results from a Minitab display. Assume that you want to use a 0.05 significance level in testing the null hypothesis that the different samples come from populations with the same mean.

5) Find the critical value.

Source	DF	SS	MS	F	p
Factor	3	13.500	4.500	5.17	0.011
Error	16	13.925	0.870		
Total	19	27.425			

- A) 3.06 B) 8.70 C) 5.42 D) 3.24

6) What can you conclude about the equality of the population means?

Source	DF	SS	MS	F	p
Factor	3	13.500	4.500	5.17	0.011
Error	16	13.925	0.870		
Total	19	27.425			

- A) Accept the null hypothesis since the p-value is less than the significance level.
 B) Reject the null hypothesis since the p-value is greater than the significance level.
 C) Reject the null hypothesis since the p-value is less than the significance level.
 D) Accept the null hypothesis since the p-value is greater than the significance level.

Test the claim that the samples come from populations with the same mean. Assume that the populations are normally distributed with the same variance.

7) A consumer magazine wants to compare the lifetimes of ballpoint pens of three different types. The magazine takes a random sample of pens of each type in the following table.

Brand 1	Brand 2	Brand 3
260	181	238
218	240	257
184	162	241
219	218	213

Do the data indicate that there is a difference in mean lifetime for the three brands of ballpoint pens? Use $\alpha = 0.01$.

- 8) At the 0.025 significance level, test the claim that the four brands have the same mean if the following sample results have been obtained.

Brand A	Brand B	Brand C	Brand D
17	18	21	22
20	18	24	25
21	23	25	27
22	25	26	29
21	26	29	35
		29	36
			37

Provide an appropriate response.

- 9) At the same time each day, a researcher records the temperature in each of three greenhouses. The table shows the temperatures in degrees Fahrenheit recorded for one week.

Greenhouse #1	Greenhouse #2	Greenhouse #3
73	71	67
72	69	63
73	72	62
66	72	61
68	65	60
71	73	62
72	71	59

- i) Use a 0.05 significance level to test the claim that the average temperature is the same in each greenhouse.
 ii) How are the analysis of variance results affected if 8° is added to each temperature listed for greenhouse #3?

Use the Minitab display to test the indicated claim.

- 10) A manager records the production output of three employees who each work on three different machines for three different days. The sample results are given below and the Minitab results follow.

		Employee		
		A	B	C
Machine	I	23, 27, 29	30, 27, 25	18, 20, 22
	II	25, 26, 24	24, 29, 26	19, 16, 14
	III	28, 25, 26	25, 27, 23	15, 11, 17

ANALYSIS OF VARIANCE ITEMS

SOURCE	DF	SS	MS
MACHINE	2	34.67	17.33
EMPLOYEE	2	504.67	252.33
INTERACTION	4	26.67	6.67
ERROR	18	98.00	5.44
TOTAL	26	664.00	

Assume that the number of items produced is not affected by an interaction between employee and machine. Using a 0.05 significance level, test the claim that the machine has no effect on the number of items produced.

- 11) A manager records the production output of three employees who each work on three different machines for three different days. The sample results are given below and the Minitab results follow.

		Employee		
		A	B	C
Machine	I	16, 18, 19	15, 17, 20	14, 18, 16
	II	20, 27, 29	25, 28, 27	29, 28, 26
	III	15, 18, 17	16, 16, 19	13, 17, 16

ANALYSIS OF VARIANCE ITEMS

SOURCE	DF	SS	MS
MACHINE	2	588.74	294.37
EMPLOYEE	2	2.07	1.04
INTERACTION	4	15.48	3.87
ERROR	18	98.67	5.48
TOTAL	26	704.96	

Using a 0.05 significance level, test the claim that the interaction between employee and machine has no effect on the number of items produced.

- 12) A manager records the production output of three employees who each work on three different machines for three different days. The sample results are given below and the Minitab results follow.

		Employee		
		A	B	C
Machine	I	31, 34, 32	29, 23, 22	21, 20, 24
	II	19, 26, 22	35, 33, 30	25, 19, 23
	III	21, 18, 26	20, 23, 24	36, 37, 31

ANALYSIS OF VARIANCE ITEMS

SOURCE	DF	SS	MS
MACHINE	2	1.19	.59
EMPLOYEE	2	5.85	2.93
INTERACTION	4	710.81	177.70
ERROR	18	160.00	8.89
TOTAL	26	877.85	

Assume that the number of items produced is not affected by an interaction between employee and machine. Using a 0.05 significance level, test the claim that the choice of employee has no effect on the number of items produced.

Use the data in the given table and the corresponding Minitab display to test the hypothesis.

- 13) The following table entries are the times in seconds for three different drivers racing on four different tracks. Assuming no effect from the interaction between driver and track, test the claim that the track has no effect on the time. Use a 0.05 significance level.

	Track 1	Track 2	Track 3	Track 4
Driver 1	72	70	68	71
Driver 2	74	71	66	72
Driver 3	76	69	64	70

Source	DF	SS	MS	F	p
Driver	2	2	1	0.33	0.729
Track	3	98.25	32.75	10.92	0.00763
Error	6	18	3		
Total	11	118.25			

- 14) The following table entries are test scores for males and females at different times of day. Assuming no effect from the interaction between gender and test time, test the claim that males and females perform the same on the test. Use a 0.05 significance level.

	6 a.m. - 9 a.m.	9 a.m. - 12 p.m.	12 p.m. - 3 p.m.	3 p.m. - 6 p.m.
Male	87	89	92	85
Female	72	84	94	89

Source	DF	SS	MS	F	p
Gender	1	24.5	24.5	0.6652	0.4745
Time	3	183	61	1.6561	0.3444
Error	3	110.5	36.83		
Total	7	318			

Provide an appropriate response.

- 15) The following data show annual income, in thousands of dollars, categorized according to the two factors of gender and level of education. Test the null hypothesis of no interaction between gender and level of education at a significance level of 0.05.

	Female	Male
High school	23, 27, 24, 26	25, 26, 22, 24
College	28, 36, 31, 33	35, 32, 39, 28
Advanced degree	41, 38, 43, 49	35, 50, 47, 44

- 16) The following data shows the yield, in bushels per acre, categorized according to three varieties of corn and three different soil conditions. Assume that yields are not affected by an interaction between variety and soil conditions, and test the null hypothesis that variety has no effect on yield. Use a 0.05 significance level.

	Plot 1	Plot 2	Plot 3
Variety 1	156, 167, 170, 162	162, 160, 169, 168	145, 151, 148, 155
Variety 2	172, 176, 166, 179	179, 186, 160, 176	161, 162, 165, 170
Variety 3	175, 157, 179, 178	178, 170, 172, 174	169, 165, 170, 169

- 17) The following data contains task completion times, in minutes, categorized according to the gender of the machine operator and the machine used.

	Male	Female
Machine 1	15, 17	16, 17
Machine 2	14, 13	15, 13
Machine 3	16, 18	17, 19

The ANOVA results lead us to conclude that the completion times are not affected by an interaction between machine and gender, and the times are not affected by gender, but they are affected by the machine. Change the table entries so that there is no effect from the interaction between machine and gender, there is no effect from the machine used, and there is no effect from the gender of the operator.

- 18) The following data contains task completion times, in minutes, categorized according to the gender of the machine operator and the machine used.

	Male	Female
Machine 1	15, 17	16, 17
Machine 2	14, 13	15, 13
Machine 3	16, 18	17, 19

Assume that two-way ANOVA is used to analyze the data. How are the ANOVA results affected if the times are all doubled?

- 19) **Provide an appropriate response.**

What advantage do confidence intervals offer to one-way ANOVA?

- 20) **Provide an appropriate response.**

What is the difference between the "omnibus" F test used here for one-way ANOVA and the homogeneity F test used in Chapter 9?

Answer Key

Testname: CHAPTER 12 FORM B

- 1) The six paired hypotheses are $\mu_A = \mu_B$, $\mu_A = \mu_C$, $\mu_A = \mu_D$, $\mu_B = \mu_C$, $\mu_B = \mu_D$, $\mu_C = \mu_D$.
Suppose we test each with a 5% significance level (95% confidence level). Then, the degree of confidence for all six would be $(0.95)^6$ or 0.735, yielding an excessively high risk of a type I error. ANOVA maintains the 5% significance level while testing equivalence of all four
- 2) A treatment (also known as a factor) is a property or characteristic that allows us to distinguish the different populations from one another. Examples will vary. Possible example: Engine type distinguished according to three models is a factor, whose associated experiment is to determine fuel efficiency.
- 3) Variation within each cell of a two-way design does not exist when there is only one observation per cell. Therefore, the measures dependent on cell variation, such as the mean sum of squares, cannot be calculated for an interaction between the two variables.
- 4) Large sample sizes coupled with substantial variation between sample means could cause a significant F ratio, especially if the pooled variation within samples is not exceedingly high.
- 5) D
- 6) C
- 7) Test statistic: $F = 1.620$. Critical value: $F = 8.0215$. P-value: $p = 0.251$.
Fail to reject the claim of equal means. The data do not provide sufficient evidence to conclude that there is a difference in the mean lifetimes of the three brands of ballpoint pen.
- 8) $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$. $H_1 : \text{The means are not all equal}$. P-value: $p = 0.00285$.
Test statistic: $F = 6.69863$. Critical value: $F = 3.9034$.
Reject the null hypothesis. There is sufficient evidence to warrant rejection of the claim that the four brands have the same mean.
- 9) i) Reject the claim that the average temperature is the same in each greenhouse since $F = 24.2899 > F_{0.05}(2, 18) = 3.5546$.
ii) Accept the claim that the average temperature is the same in each greenhouse since $F = 0.12809 < F_{0.05}(2, 18) = 3.5546$.
- 10) H_0 : There is no machine effect.
 H_1 : There is a machine effect.
Test statistic: $F = 3.1857$. Critical value: $F = 3.5546$.
Fail to reject the null hypothesis. The type of machine does not appear to have an effect on the number of items produced.
- 11) H_0 : There is no interaction effect.
 H_1 : There is an interaction effect.
Test statistic: $F = 0.7062$. Critical value: $F = 2.9277$.
Fail to reject the null hypothesis. There does not appear to be an interaction effect.
- 12) H_0 : There is no employee effect.
 H_1 : There is an employee effect.
Test statistic: $F = 0.3296$. Critical value: $F = 3.5546$.
Fail to reject the null hypothesis. There does not appear to be an employee effect.
- 13) H_0 : There is no track effect. H_1 : There is a track effect. The P-value is 0.00763, which is less than 0.05. We reject the null hypothesis; it appears that the track does effect the racing times.
- 14) H_0 : There is no gender effect. H_1 : There is a gender effect. The P-value is 0.4745, which is greater than 0.05. We fail to reject the null hypothesis; it appears that the scores are not affected by gender.

Answer Key

Testname: CHAPTER 12 FORM B

- 15) H_0 : There is no interaction between gender and level of education. H_1 : There is an interaction between gender and level of education. The test statistic is $F = 0.177472$, and the corresponding P-value is 0.838832. Because the P-value is greater than 0.05, we fail to reject the null hypothesis of no interaction between gender and level of education.
- 16) H_0 : Variety has no effect on yield
 H_1 : Variety has an effect on yield
The test statistic is $F = 13.54801$, and the corresponding P-value is 0.0000843. Because the P-value is less than 0.05, we reject the null hypothesis that variety has no effect on yield. It appears that the variety of corn does affect the yield.
- 17) The following table is one example of entries that produce no effect from interaction between gender and machine, no effect from machine, and no effect from gender.

	Male	Female
Machine 1	15, 17	16, 17
Machine 2	17, 13	15, 13
Machine 3	16, 18	17, 19

- 18) The ANOVA results are not affected by doubling the completion times. The null hypothesis of no interaction between machine and gender is not rejected since the P-value is 0.946. The null hypothesis of no effect from machine is rejected since the P-value is 0.013. The null hypothesis of no effect from gender is not rejected since the P-value is 0.382.
- 19) After a significant F test, confidence intervals of the means of data sets can be used as an informal method to judge non-overlap, which suggests differences among means.
- 20) The "omnibus" F test applied in one-way ANOVA outputs the overall F ratio, based on variance of the means, contributed by three or more samples. This is a hypothesis test of equality of means of the populations from which the samples were selected. On the other hand, the homogeneity of variance test in Chapter 9 is a hypothesis test that two populations have the same variance, based on their respective sample variances. (This latter test may be used to test the assumption, i.e. requirement, that two populations have equal variances, prior to a hypothesis test.)