Assignment NO. 2 week4-week7

Student Full Name:_____ .

Student ID:_____ .

CRN No:_____ .

Branch: _____.

STATISTICS (STAT-101)	

Total Points

True/False	____/6
MCQ	____/6
Short Answer	____/18
Total	____/30

Good Luck

STATISTICS (STAT-101)

Marks- 30

Answer all the Questions on the same question paper.

Section-I

State whether the following statements are True or False. (6 marks, *1 Mark Each)*

1. The probability of an impossible event is one. (FALSE)
2. P(B|A) = P(Event A occurs in a first trial and event B occurs in a second trial). (FALSE)
3. The count of the number of students present in class on a given day is a discrete random variable. (TRUE)
4. The distribution of sample $\bar{x}$ will, as the sample size increases, approach a normal distribution. (TRUE)
5. The Student t distribution is different for different sample sizes. (TRUE)
6. It is 95% (or 0.95) confidence level for corresponding value of $\alpha = 0.1$ (FALSE)

Section-II

(Multiple Choice Questions) *(*6 marks, *1 Mark Each)*

1. In a box, there are 8 red, 7 blue and 6 green balls. One ball is picked up randomly. What is the probability that it is neither red nor green?
   a. $\frac{1}{3}$
   b. $\frac{1}{21}$
   c. $\frac{7}{21}$
   d. $\frac{8}{21}$

2. Which one of these variables is a continuous random variable?
   a. The time it takes a randomly selected student to complete an exam
   b. The number of shareholders of a randomly selected company

c. The number of women taller than 68 inches in a random sample of 5 women.
d. The number of correct guesses in a multiple choice test.

3. A random variable x follows the Binomial distribution with n=4 and p = 0.6; P(x=3)=
   a. 0.013
   b. 0.24
   c. 0.3456
   d. 0.96

4. If the z – score of normal distribution is 2.5, the mean of the distribution is 45 and the standard deviation of normal distribution is 3 then the value of X for a normal distribution is:
   a. 97.5
   b. 47.5
   c. 52.5
   d. 67.5

5. A distribution of data that is the left half of its histogram is roughly a mirror image of its right half called:
   a. Skewed
   b. Symmetric
   c. both a and b
   d. None of the above

6. Express the 95% confidence interval of $-0.001 < p < 0.559$ in the form $\hat{p} \pm E$.
   a. 0.28 ± 0.279
   b. 0.279 ± 0.5
   c. 0.279 ± 0.28
   d. 0.28 ± 0.5

Section-II (Multiple Choice Questions) *(*6 marks, 1 Mark Each)

MCQ	1	2	3	4	5	6
Answers						

Section –III

Answer the following Essay Type Questions (18 marks, *3 Mark Each*)

1. Consider the following (hypothetical) distribution of gender and major of students in an introductory statistics class:

	social science	non-social science	total
female	30	20	50
male	30	20	50
total	60	40	100

   a. Find the probability that a randomly selected student is a social science major.
   b. Find the probability that a randomly selected student is a social science major given that they are female.

Answer:

   a. The total number of students in social science = 60
      The total number of students = 100
   So, The probability that a randomly selected student is a social science major = 60 / 100 = 0.6
   c. The number of female = 50
      The number of social science **female** students = 30
   So P(Social science | female) = 30 / 50 = 0.6.

2. The random variable X has the following probability distribution:

X	0	1	2	3
P(x)	0.22	0.38	0.1	0.3

   a. Prove that the given table satisfies the two properties needed for a probability distribution
   b. Find the mean & the Variance.

Solution:

Sum of the probabilities =1

Each probability is between 0 and 1

So, the table satisfies the two properties needed for a probability distribution

X	0	1	2	3	Total
P(x)	0.22	0.38	0.1	0.3	1
$x.p(x)$	0	0.38	0.2	0.9	1.48

x^2	0	1	4	9	14
$x^2.p(x)$	0	0.38	0.4	2.7	3.48

Mean = $E(x) = \sum x.p(x) = 1.48$

Variance = $V(x) = E(x^2) - \{E(x)\}^2 = \sum x^2.p(x) - \{1.48\}^2 = 3.48 - 2.19 = 1.29$

3. If X is binomially distributed with 6 trials and a probability of success equal to ¼ at each attempt. What is the probability of exactly 4 success? Also find the parameter mean, variance, and standard deviation.

Answer: Here n = 6, p = ¼ = 0.25, x = 4, q = 0.75
Then P(x = 4) = 0.03296
Mean: μ= np = 6 × 0.25 = 1.50
Variance: σ^2 = n p q = 6 × 0.25 × 0.75 = 1.125
Standard Deviation: σ = √ 1.125 = 1.061

4. Z follows a standard normal distribution. Find
a) P(Z < 2.37)
b) P(Z > 1.82)
c) P(-1.18 < Z < 2.1)

Answer: We use the table.

a) P(z < 2.37) = .9911

b) P(z > 1.82) = 1 - P(z < 1.82)
P(z < 1.82) = .9656
P(z > 1.82) = 1 - .9656 = .0344

c)reading the table gives
P(z < 2.10) = .9821.
And P(z< -1.18) = 0.1190
Hence
P(-1.18 < z < 2.1) = P(z < 2.1) - P(z < -1.18)
= .9821 - .1190 = .8631

5. There are 250 dogs at a dog show who weigh an average of 12 pounds, with a standard deviation of 8 pounds. If 4 dogs are chosen at random, what is the probability they have an average weight of greater than 8 pounds and less than 25 pounds?

Answer. Use the following formula to find the z-scores.

$$z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$

here μ= 12, σ= 8 , n = 4, we have to calculate Z-score between average weight of greater than 8 pounds and less than 25 pounds.

$= \left(\frac{8-12}{8/\sqrt{4}} < z < \frac{25-12}{8/\sqrt{4}} \right)$

$= (-1 < z < 3.25)$

$= (z < 3.25) – (z < -1)$

= see value by standard normal distribution table

= 0.9994 – 0.1587 = 0.8407

6. Suppose a student measuring the boiling temperature of a certain liquid observes the readings (in degrees Celsius) 102.5, 101.7, 103.1, 100.9, 100.5, and 102.2 on 6 different samples of the liquid. He calculates the sample mean to be 101.82. If he knows that the standard deviation for this procedure is 1.2 degrees, what is the confidence interval for the population mean at a 95% confidence level?

Answer:

The student wishes to estimate the true mean boiling temperature of the liquid using the results of his measurements. If the measurements follow a normal distribution, then the sample mean will have the distribution $N(\mu, \frac{\sigma}{\sqrt{n}})$. Since the sample size is 6, the standard deviation of the sample mean is equal to 1.2/sqrt(6) = 0.49.

For a population with unknown mean μ and known standard deviation σ, a confidence interval for the population mean, based on a simple random sample (SRS) of size *n*, is $\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}}$, where z^* is the critical value for the standard normal distribution.
The critical value for a 95% confidence interval is 1.96.
A 95% confidence interval for the unknown mean μ is ((101.82 - (1.96*0.49)), (101.82 + (1.96*0.49))) = (101.82 - 0.96, 101.82 + 0.96) = (100.86, 102.78).