

Assignment 3 week 9-week 11

Student Full Name: _____.

Student ID: _____.

CRN No: _____.

Branch: _____.

STATISTICS (STAT-101)

Total Points

True/False _____/6

MCQ _____/6

Short Answer _____/18

Total _____/30

Good Luck

STATISTICS (STAT-101)

Marks- 30

Answer all the Questions on the same question paper.

Section-I

State whether the following statements are True or False. (6 marks, 1 Mark Each)

1. A type I error is the mistake of rejecting the null hypothesis when it is actually false. **False**
2. Two samples are independent if the sample values selected from one population are not related to the sample values from the other population. **False True**
3. The null hypothesis (denoted by H_0) is a statement that the value of a population parameter is equal to some value. **True**
4. In case of hypothesis testing for a sample, the t statistic is used if σ is not known and sample size n is greater than 5 ($n > 5$) **False**
5. A claim that two population proportions are equal, each of the two samples must satisfy the requirement that $np \geq 30$ and $nq \geq 30$. **False**
6. In an unpaired samples t-test with sample sizes $n_1 = 21$ and $n_2 = 11$, the value of t should be obtained at 32 degree of freedom. **False**

Section-II

(Multiple Choice Questions)

(6 marks, 1 Mark Each)

1. If $p\text{-value} < \alpha$, then
 - A. Reject H_0
 - B. Accept H_0
 - C. Reject H_1
 - D. All the above
2. The MEAN of the Student t distribution is
 - A. equal to 0
 - B. less than 1
 - C. less than 0
 - D. greater than 1

3. A decision in a hypothesis test can be made by using a :
 1. P-value
 2. Critical Value
 3. A and B
 4. None of the above

4. When carrying out a large sample test of $H_0: \mu_0 = 10$ vs. $H: \mu_0 > 10$ by using a rejection point, we reject H_0 at level of significance α when the calculated test statistic is:
 1. Less than z_α
 2. Less than $-z_\alpha$
 3. Greater than $z_{\alpha/2}$
 4. Greater than z_α

5. A randomly selected sample of 500 college students was asked whether they had ever used the drug Ecstasy. Sixteen percent (16% or 0.16) of the 500 students surveyed said they had. Which one of the following statements about the number 0.16 is correct?
 - A. It is a population proportion.
 - B. It is a margin of error.
 - C. It is a sample proportion.
 - D. It is a randomly chosen number.

6. Sample sizes $n_1 = 100$, $n_2 = 100$ and numbers $x_1 = 39$, $x_2 = 41$ of successes to find the pooled estimate \bar{p}
 1. 0.4
 2. 0.8
 3. 0.36
 4. 0.48

Part-II (Multiple Choice Questions)

(6 marks, 1 Mark Each)

MCQ	1	2	3	4	5	6
Answers	A	A	C	D	C	A

Section –III

Answer the following Essay Type Questions

(18 marks, 3 Mark Each)

1. Suppose the national unemployment rate is 3%. In a survey of $n = 450$ people in a rural Wisconsin county, 22 people are found to be unemployed. County officials apply for state aid based on the claim that the local unemployment rate is higher than the national average. Test this claim at the .05 significance level.

Requirements are satisfied: simple random sample; fixed number of trials (450) with two categories (employed or unemployed);
 $np = (450)(0.03) = 135 \geq 5$ and $nq = (450)(0.97) = 436.5 \geq 5$

Step 1: Write the null and alternative hypotheses

$$H_0: p = 0.03$$

$$H_1: p > 0.03$$

Step 2: test statistics

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{22}{450} - 0.03}{\sqrt{\frac{(0.03)(0.97)}{450}}}$$

$$= 2.33$$

(2.349)

Step 3:

Compare the z-score of 2.35 to the critical value for a right-tailed test at the .05 level (1.645) to determine if it's significant at the .05 level.

Since 2.33 is greater than 1.645, this means we reject the null hypothesis.

The sample data support the claim that this particular county has an unemployment rate higher than the national average.

2. Suppose we would like to determine if the typical amount spent per customer for dinner at a new restaurant in town is more than \$20.00. A sample of 49 customers over a three-week period was randomly selected and the average amount spent was \$22.60. Assume that the standard deviation σ is known to be \$7.50. Using a 0.05 level of significance, would we conclude the typical amount spent per customer is more than

\$20.00?

σ is known (\$7.50), sample size is 49 ($n > 30$)

Step 1

The claim is $\mu > 20$

Step 2

Alternative to claim is

$\mu \leq 20$

Step 3

The null and alternative hypothesis are:

$H_0: \mu = 20, H_a: \mu > 20$

Step 4

the significance level is $\alpha = 0.05$

Step 5

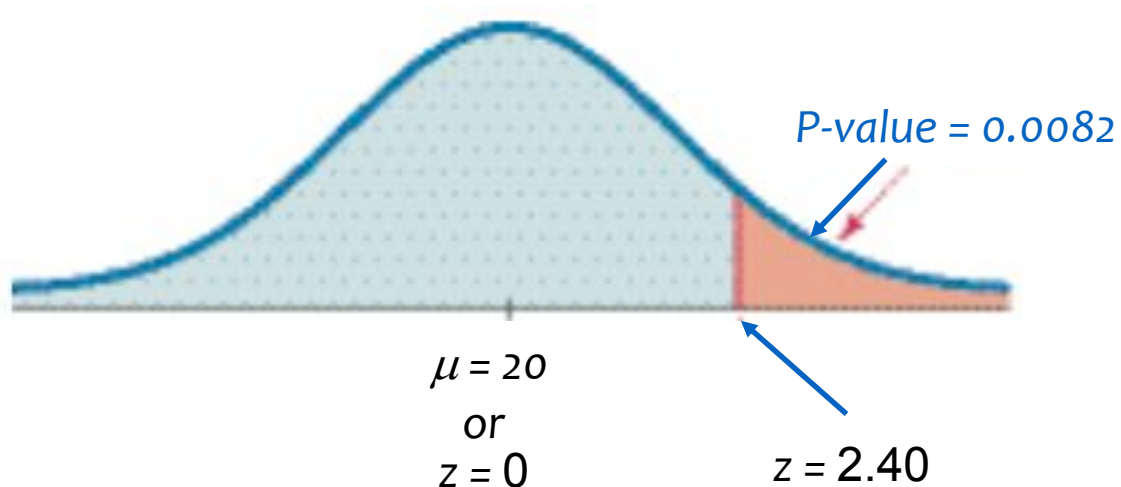
$$\sigma = 7.5, n = 49$$
$$\bar{x} = 22.60, \mu = 20$$

Step 6:

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{22.60 - 20}{\frac{7.50}{\sqrt{49}}} = 2.4$$

Right-tailed test, so P-value is the area is to the right of $z = 2.4$; The P-value of 0.0082 is less than the significance level of $\alpha = 0.05$, reject the null Hypothesis.

There is sufficient evidence to conclude the typical amount spent per customer is more than \$20.00.



3. The scores on an aptitude test required for entry into a certain job position have a mean at most 500. If a random sample of 36 applicants have a mean of 546 and a standard deviation of 120, is there evidence that their mean score is different from the mean that is expected from all applicants?. Use a 0.05 level of significance.

Null and Alternative Hypothesis

$$H_0 : \mu = 500$$

$$H_A : \mu \neq 500$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{546 - 500}{\frac{120}{\sqrt{36}}} = \frac{46}{20} = 2.3$$

This means that 546 is 2.3 standard deviations from the hypothesized mean.

Using the t-table, we find that the probability is 2.030

By comparing critical t and calculated t

$$t_{critical} = 2.030 < t_{calculated} = 2.3$$

\therefore we reject the null hypothesis.

Thus, if the mean is really 500, it is unlikely that we would get a sample mean that is 2.3 standard deviations from it. Thus, we conclude that the population mean is not 500; that is we reject the null hypothesis and accept the alternate, concluding that the mean is not 500.

4. The table show the number satisfied in their work in a sample of working adults with a college education and in a sample of working adults without a college education. Do the data provide sufficient evidence that a greater proportion of those with a college education are satisfied in their work? Use a significance level of $\alpha = 0.05$ to test the claim that $p_1 < p_2$.

	College Education	No College Education
Number satisfied in their work	12	27
Number in sample	46	43

Step 1

The null and alternative hypothesis are:

$$H_0: P_1 = P_2.$$

$$H_a: P_1 < P_2 \text{ (Claim)}$$

Step 2

significance level is $\alpha = 0.05$

Step 3

We use the normal distribution as an approximation to the binomial distribution.

Estimate the common value of p_1 and p_2

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{12 + 27}{46 + 43} = 0.4382,$$

$$\bar{q} = 0.5617$$

$$\bar{p}\bar{q} = (0.4382 \times 0.5617) = 0.2461$$

Step 4

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}}$$
$$z = \frac{\frac{12}{46} - \frac{27}{43} - 0}{\sqrt{\frac{0.4382 \times 0.5617}{46} + \frac{0.4382 \times 0.5617}{43}}} = \frac{-0.3670}{0.1052} = -3.4885$$

For the critical values in this left – tailed test by table A – 2 for the test statistics $z = -3.4885$, area is 0.0002 from the left tail. So the p-value is 0.0002.

Since the p-value is less than the significance level $\alpha = 0.05$,

So we reject the null hypothesis of $p_1 = p_2$

Q. 5 & 6. Use the following information to answer Questions 5 and 6:

Given the following data of two independent samples of normally distributed populations

Data	Population 1	Population 2
n	23	13
\bar{x}	43	41
s	4.5	5.1

- A. Test the claim that $\mu_1 \neq \mu_2$ at the $\alpha=0.05$ level of significance
B. Construct a 95% confidence interval about $\mu_1 - \mu_2$.

Solution(5) : two samples are independent and

$$\bar{x}_1 = 43, n_1 = 23, s_1 = 4.5.$$

$$\bar{x}_2 = 41, n_2 = 13, s_2 = 5.1.$$

Consider

μ_1 : mean of population 1

μ_2 : mean of population 2

Step 1

The claim is $\mu_1 \neq \mu_2$

Step 2

Alternative to claim is $\mu_1 = \mu_2$

Step 3

The null and alternative hypothesis are:

$$H_0: \mu_1 = \mu_2.$$

$$H_a: \mu_1 \neq \mu_2 \text{ (Claim)}$$

Step 4

Significance level is $\alpha = 0.05$

Step 5

$$\bar{x}_1 = 43, n_1 = 23, s_1 = 4.5.$$

$$\bar{x}_2 = 41, n_2 = 13, s_2 = 5.1.$$

Step 6:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$t = \frac{43 - 41}{\sqrt{\frac{4.5 \times 4.5}{23} + \frac{5.1 \times 5.1}{13}}} = 1.178$$

The t- value at $\alpha/2 = 0.025$ and degree of freedom $df=n-1=13-1=12$ is ± 2.179 .

Because the test statistic does not fall within the critical region, fail to reject the null hypothesis: $\mu_1 = \mu_2$

Solution(6): Margin of error (E) is given by

$$E = t_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Thus

$$E = 2.179 \sqrt{\frac{4.5 \times 4.5}{23} + \frac{5.1 \times 5.1}{13}} = 2.179 \times 1.697 = 3.699$$

Confidence interval is given by

$$(\bar{x}_1 - \bar{x}_2) - E < (\mu_1 - \mu_2) < (\bar{x}_1 - \bar{x}_2) + E$$

$$(43 - 41) - 3.699 < (\mu_1 - \mu_2) < (43 - 41) + 3.699$$

$$-1.699 < (\mu_1 - \mu_2) < 5.699$$

Since the CI has the zero, so we again conclude that we fail to reject the null hypothesis.