Assignment No. 2 week5-week7

Student Full Name:_________________________________ .

Student ID:____ .

CRN No:____ .

Branch: ____.

STATISTICS (STAT-101)

Total Points

True/False ____/6

MCQ ____/6

Short Answer ____/18

Total ____/30

Good Luck

STATISTICS (STAT-101)

Marks- 30

Answer all the Questions on the same question paper.

Section-I

State whether the following statements are True or False. (*6 marks, 1 Mark Each)*

1. The Student t distribution is same for different sample sizes.	**F**
2. The distribution of sample $\bar{x}$ will, as the sample size increases, approach a normal distribution. .	**T**
3. For any normal distribution, the mean, median, and mode will have the same value.	**T**
4. The Central Limit Theorem tells us that as the sample size *n* increases, the sampling distribution of the sample mean will approach a binomial distribution.	**F**
5. If the z – score of normal distribution is 2, the mean of the distribution is 30 and the standard deviation of normal distribution is 9 then the value of X for a normal distribution is 48	**T**
6. The time it takes a randomly selected student to complete an exam is discrete random variable	**F**

Section-II

Mark the right answer from following multiple choice questions:

(Multiple Choice Questions) (6 marks, *1 Mark Each)*

1. A police department reports that the probabilities that 0, 1, 2 and 3 burglaries will be reported in a given day are 0.45, 0.44, 0.09 and 0.02, respectively. Find the standard deviation for the probability distribution. Round answer to the nearest hundredth.
   a. $\sigma = 0.52$
   b. $\sigma = 0.72$
   c. $\sigma = 0.99$
   d. $\sigma = 0.98$

2. A distribution of data that is the left half of its histogram is roughly a mirror image of its right half called:
   a. Skewed
   b. Symmetric
   c. Both a and b
   d. None of the above

3. Which of the following is not a property of a binomial experiment?
   a. The experiment consists of *n* identical trials.
   b. The trials are independent, that is, the outcome of one trial does not influence the outcome of another.
   c. The two outcomes, success (S) and failure (F) are equally likely to occur.
   d. The probability of a success on a single trial is equal to p, and p remains constant from trial to trial.

4. The following confidence interval is obtained for a population proportion *p*: [0.528 , 0.554]. The margin of error *E* is equal to
   a. 0.013
   b. 0.026
   c. 0.011
   d. 0.014

5. The critical value $z_{\alpha/2}$ that corresponds to a 90% confidence level is
   a. 1.340
   b. 1.700
   c. 1.645
   d. 1.750

6. An industrial designer wants to determine the average amount of time it takes an adult to assemble an "easy to assemble" toy. A sample of 16 times yielded an average time of 19.92 minutes, with a sample standard deviation of 5.73 minutes. Assuming normality of assembly times, provide a 95% confidence interval for the mean assembly time.
   a. 19.92 ± 2.81
   b. 19.92 ± 3.05
   c. 19.92 ± 3.04
   d. 19.92 ± 2.51

MCQ	1	2	3	4	5	6
Answers	b	b	c	a	c	b

Section –III

Answer the following Essay Type Questions (18 marks, *3 Mark Each*)

1. X follows a normal distribution with mean μ=2 and variance $\sigma^2 = 9$ and Z denotes the corresponding standard normal distribution. Find
   a) $P(X < 2)$
   b) $P(Z > 1)$
   c) $P(-2 < X < 2)$

Solution:

a.

$$\boldsymbol{P(X < 2) = P[(X - \mu)/\sigma < (2 - \mu)/\sigma]}$$

$$= P[Z < 0] = 0.5$$

b.

$$P(Z > 1) = 1 - P[Z < 1] = 1 - 0.8413447 = 0.1586553$$

c.

$$P(-2 < X < 2) = P[(-2 - \mu)/\sigma < (X - \mu)/\sigma < (2 - \mu)/\sigma]$$

$$= P[-1.33 < Z < 0] = P[Z < 0] - P[Z < -1.33]$$

$$= 0.5 - 0.09176 = 0.40824$$

2. Twelve percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 10 parts contains

   a. No defective ones (i.e. $P(X=0)$)
   b. More than 3 defective ones (i.e. $P(X>3)$)

Solution:

We need to find $P(X=0)$ and $P(X > 3)$, where X is the number of defective parts in a sample of 10 parts. This X is the number of "successes" in 10 trials, therefore, it has Binomial distribution with parameters $n = 10$ and $p = 0.12$.

Solution

a. $P(X = 0) = (0.88)^{10} = 0.2785$

b. $P(X > 3) = 1 - P(0) - P(1) - P(2) - P(3)$
$= 1 - (0.88)^{10} - (10)(0.12)(0.88)^9$
$- (10\ \ (9/2))(0.12)^2 (0.88)^8 - (10 \cdot 9 \cdot (8/6))(0.12)^3 (0.88)^7$
= 1-0.2785-0.3798-0.2330-0.0847
=0.0240.

3. Given that, there is a 0.8 probability that a randomly selected adult knows what Facebook is, use the binomial probability formula to find, when 3 adults are randomly selected:

   A. $P(X=2)$, i.e., 2 adults who know what Facebook is

B. *P(X at least 1)*, i.e., at least 1 adult who knows what Facebook is
C. *P(X at most 2), i.e.,* At most1 adult who knows what Facebook is

Solution:

Using the given values of $n = 3$, x, $p = 0.8$, and $q = 0.2$ in the binomial probability formula, we get

a. $P(X=2) = \frac{3!}{(3-2)!2!}.0.8^2.0.2^{3-2}$

$= \frac{3!}{1!2!} \times 0.64 \times 0.2$

$= 0.384$

b. $P(X \text{ at least } 1) = 1 - P(X = 0) = 1 - 0.008 = 0.992$

c.
$P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$
$= 0.008 + 0.096 + 0.384 = 0.488$

4. Suppose that you have a sample of 121 values from a population with mean $\mu = 500$ and with standard deviation $\sigma = 88$. What is the probability that the sample mean will be in the interval (490, 510)?

Solution:

We are given $\mu=500$ and $\sigma=88$ and $n=121$. We know from the Central Limit Theorem that

$$\bar{X} \sim Normal(\mu, (\sigma^2)/n).$$

This exercise requires that we know how to transform $\bar{X}$ into Z. To transform $\bar{X}$ into Z we use

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Then:

$$P(490 < \overline{X} < 510) = P\left(\frac{490-500}{\frac{88}{\sqrt{121}}} < Z < \frac{510-500}{\frac{88}{\sqrt{121}}} \right) = P(-1.25 < Z < 1.25)$$

Now, if we look into a Normal Distribution table:
$P(-1.25<Z<1.25) = 0.8944 - (1-0.8944)$
= 0.7888 =78.88%

5. The operations manager of a large production plant would like to estimate the mean amount of time a worker takes to assemble a new electronic component. Assume that the standard deviation of this assembly time is 3 minutes. After observing 100 workers assembling similar devices, the manager noticed that their average time was 16 minutes. Construct a 90% confidence interval for the mean assembly time.

Solution:

Let μ denote the mean assembly time (in minutes). It is given that σ = 3 min. We want a 90% confidence interval for μ based on the following information: $n = 100$, $\overline{X}$ = 16 min, α = 1 - 0.90 = 0.1, and σ = 3 min. Since σ is known, we will use the z-interval:

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 16 \pm 1.645\frac{3}{\sqrt{100}}$$

=[16-0.4935 , 16+0.4935]

=[15.51 ,16.49]

where $z_{0:05}$ = 1.645 obtained from the normal table. Thus, the mean assembly time for a worker is estimated to be between 15.51 min and 16.49 min, with 90% confidence.

6. The random variable X has the following probability distribution:

X	0	1	2	3
P(x)	0.2	0.1	0.4	*K*

a. Find *K* to Prove that the given table satisfied a probability distribution
b. Find the mean & the Variance.

Solution:

a. $\sum(P(X = x)) = 0.2 + 0.1 + 0.4 + K = 1$

this implies that

$K = 1 - 0.7 = 0.3$

X	0	1	2	3	Sum
$P(x)$	0.2	0.1	0.4	0.3	1
$x\,P(X = x)$	0	0.1	0.8	0.9	1.8
x^2	0	1	4	9	
x^2 P(X=x)	0	0.1	1.6	2.7	4.4

b.

Mean: $E(X) = \sum xP(X = x) = 1.8$

$$E(X^2) = \sum x^2 P(X = x) = 4.4$$

Variance: $V(X) = E(X^2) - [E(X)]^2$

$= 4.4 - 3.24 = 1.16$