Linear Programming Problem
Week-14

LPP : Maximizing or minimizing a linear function under some linear constraints is called LPP.

Mathematical formulation of LPP

Maximizing $Z = C' \underline{x}$ (transpose) $\longrightarrow$ Objective function

Subject to $A\underline{x} \leq b$ $\longrightarrow$ Constraints

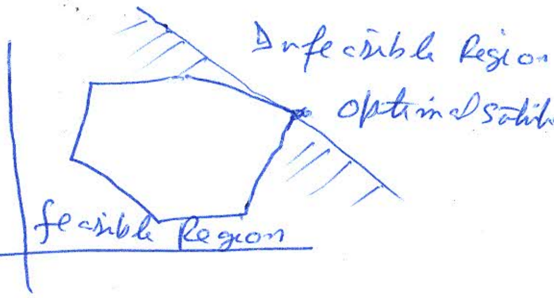
$b, x \geq 0$ $\longrightarrow$ non negative restriction

where

$$c' = \begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}, \quad x = (x_1 -- x_n), \quad A = \begin{pmatrix} a_{11} \cdots a_{1n} \\ a_{m1} \cdots a_{mn} \end{pmatrix}$$

We can solve LPP by Three methods

(i) Graphical Method (ii) Simplex method
(iii) Dual Simplex Method



$\Rightarrow$ In Simplex Method we move from one feasible point to another feasible point and reach the optimal solution

In dual Simplex we move from infeasible ~~to~~ Region to optimal solution

Q. Using Graphical Method to solve

$$\text{Max} \quad Z = 3x_1 + 2x_2 \quad \text{—(1)}$$

Subject to

$$x_1 + x_2 \leq 4 \quad \text{—(2)}$$

$$x_1 - x_2 \leq 2 \quad \text{—(3)}$$

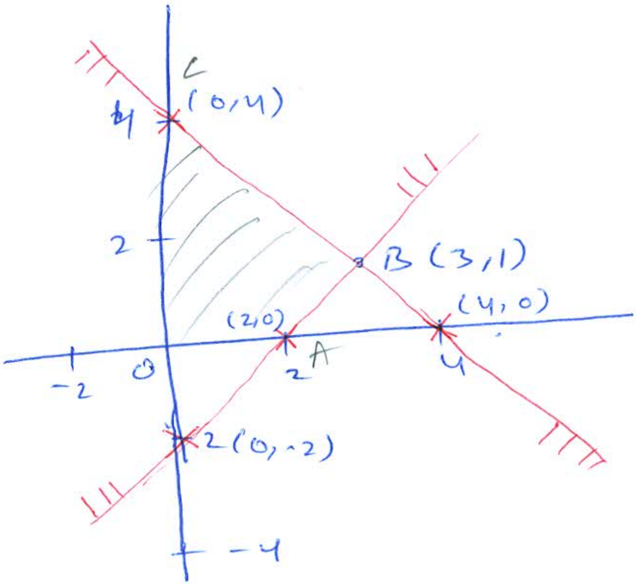
$$(x_1, x_2) \geq 0$$

Soln: for equation (2)

$x_1 + x_2 = 4$

x_1	0	4
x_2	4	0

Since $0 + 0 \leq 4$ true
feasible Region lie on the origin



for equation (3)

$x_1 - x_2 = 2$

x_1	0	2
x_2	-2	0

Since $0 - 0 \leq 2$ true
$\therefore$ origin lie in the feasible Region

Now find the coordinate of B

$$\begin{aligned} x_1 + x_2 &= 4 \\ -x_1 \mp x_2 &= -2 \\ \hline 2x_2 &= 2 \end{aligned}$$

$x_2 = 1$ put in (1) $x_1 + 1 = 4$
$x_1 = 3$

Now Draw the table

S.No	Corner Points	Value of $Z = 3x_1 + 2x_2$
1	A (2,0) ~~(0,-2)~~	$3 \times 2 + 2 \times 0 = 6$
2	B (3,1)	$3 \times 3 + 2 \times 1 = \boxed{11}$
3	C (0,4)	$3 \times 0 + 2 \times 4 = 8$
4	O (0,0)	$3 \times 0 + 2 \times 0 = (0,0)$

The maximum value of Z occurs at B (3,1)
hence the optimal solution is $x_1 = 3$, $x_2 = 1$

Use Simplex method to solve the LPP

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject to
$$x_1 + x_2 \le 4$$
$$x_1 - x_2 \le 2$$
$$(x_1, x_2) \ge 0$$

Soln: We solve this problem in 4 steps

Step 1: Write in standard form by introducing a slack & surplus variable

$$\text{Max } Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$$

subject to
$$x_1 + x_2 + S_1 = 4$$
$$x_1 - x_2 + S_2 = 2$$
$$(x_1, x_2, S_1, S_2) \ge 0$$

Step 2: Since we have 4 ~~eqn~~ variable and 2 equations so $4 - 2 = 2$ variable equal to zero, let $x_1 = x_2 = 0$ so $S_1 = 4$, $S_2 = 2$

Step 3: Draw the table

CB	Basis	Solution c_j	x_1 (3)	x_2 (2)	S_1 (0)	S_2 (0)	Ratio
0	S_1	4	1	1	1	0	$4/1 = 4$
0	S_2	2	1	-1	0	1	$2/1 = 2$ ← minimum Ratio
	Z_j	6	0	0	0	0	
	$Z_j - C_j$	0	-3	-2	0	0	

Pivotal Row

Pivotal element

↑ most -ve value → Pivotal column

Step 4: check optimality : we get -ve value in $Z_j - C_j$ Row, Now draw Next table

divide the pivotal element in pivot Row.

Now apply the Row operation $R_1 \to R_1 - R_2$
$R_3 \to R_3 + 3R_2$

so that the pivotal element is 1 and above & below of pivotal element is 0.

Basis	Solution	x_1	x_2	S_1	S_2	Ratio
$R_1 \to R_1 - R_2 \to S_1$	2	0	$\boxed{2}$	1	-1	$2/2 = 1 \leftarrow$
x_1	2	1	-1	0	1	$2/-1 = -2$
$R_3 \to R_3 + 3R_2 \to$	6	0	-5 ↑	0	3	

Rough

old value − key colm entry × New key Row

~~$4 - 1 \times 2 = 2$~~
~~$1 - 1 \times 1 = 0$~~
~~$1 - 1 \times (-1) = 2$~~
~~$1 - 1 \times 0 = 1$~~
~~$0 - 1 \times 1 = -1$~~

~~$0 - (-3) \times 2 = 6$~~
~~$-2 + 3(-1) = -5$~~
~~$0 - (-3) \times 0 = 0$~~
~~$0 + 3 + 1 = 3$~~

Now Repeat the Process again

divid the whole Row by 2

		x_1	x_2	S_1	S_2	
x_2	1	0	$\boxed{1}$	$1/2$	$-1/2$	
	2	1	-1	0	1	$R_2 \to R_2 + R_1$
	6	0	-5		3	$R_3 \to R_3 + 5R_1$

Now apply the Row opration on R_2 & R_3 so that they become 0

$R_2 \to R_2 + R_1$ and $R_3 \to R_3 + 5R_1$

	Solution	x_1	x_2	S_1	S_2	Ratio
x_2	1	0	$\boxed{1}$	$1/2$	$-1/2$	
x_1	3	1	0	$1/2$	$1/2$	
	11	0	0	$5/2$	$1/2$	

Since all $Z_j - C_j \geq 0$, The Solution is optimum.

The optimal solution is max $Z = 11$, $x_1 = 3$, $x_2 = 1$

Note: Compare with the graphical method we get same solu

Solve Graphically

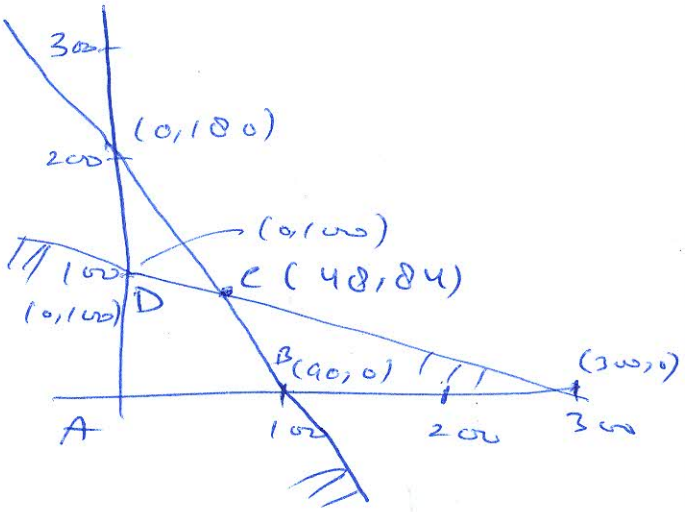
Max $Z = x + 1.2y$ —(1)

Subject to $2x + y \leq 180$ —(2)

$x + 3y \leq 300$ —(3)

$(x, y) \geq 0$

from (2)



x	90	0
y	0	180

$2 \cdot 0 + 0 \leq 180$ true

So feasible Region Lie the ...

from (3)

x	0	300
y	100	0

$0 + 3 \times 0 \leq 300$ true

So feasible in the origin

Solve (2) & (3)

$x = 48$ and $y = 84$

	Vertex	$Z = x + 1.2y$
1	A (0,0)	0
	B (90,0)	90
	C (48,84	148.8
	D (0,100)	120

Q Solve the Linear Programming Problem by graphical method

Max $Z = 50x + 18y$

subject to $2x + y \leq 100$ —(1)

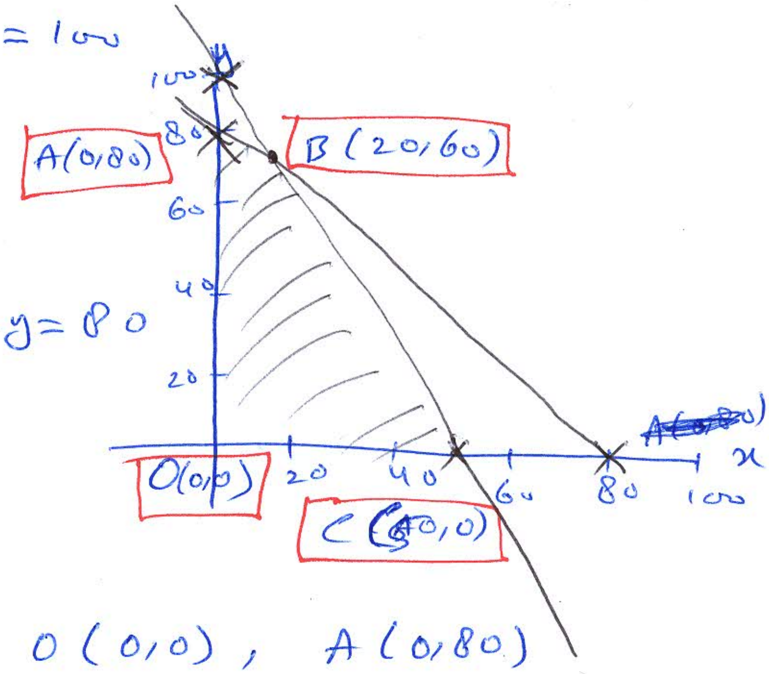
$x + y \leq 80$ —(2)

$x \geq 0, y \geq 0$

Soln: Since $x, y \geq 0$ we consider only first quadrant

Now for line (1) $2x + y = 100$

x	0	50
y	100	0



Similarly for line (2) $x + y = 80$

x	0	80
y	80	0

So the feasible Region is O(0,0), A(0,80)
B(20,60), C(50,0)

Points	$Z = 50x + 18y$
O(0,0)	0
A(0,80)	1440
B(20,60)	2080
C(50,0)	2500

Since our Object is to maximize z So the optimal solution is
$x = 50$ and $y = 0$
The Optimal value is 2500