Ministry of Higher Education
Kingdom of Saudi Arabia

CSTS
SEU, KSA

Linear Algebra (Math 251)
Level IV, Assignment 2
(2016)

1. State whether the following statements are true or false: [6]

   (a) If (-2,3) and (4,1) are the initial and terminal points respectively then (-2,2) is the components of the vector.

   (a) False

   (b) If $\theta = 180°$, be the angle between two vectors then these vectors are orthogonal.

   (b) False

   (c) The set $\mathbb{R}^3$ is a subspace of $\mathbb{R}^4$.

   (c) False

   (d) The set $\{(1,2,1),(0,1,4),(6,12,6)\}$ of vectors in $\mathbb{R}^3$ is linearly dependent.

   (d) True

   (e) The basis of a vector space is not unique.

   (e) True

   (f) If A is a 3×3 matrix such that $|A| \neq 0$ then row vectors of A span $\mathbb{R}^3$.

   (f) True

2. Select one of the alternatives from the following questions as your answer. [6]

   (a) If $\|u\| = 1$, $\|v\| = 4$ and $u.v = 0$, then the angle between u and v is

      A. $\frac{\pi}{4}$
      B. $\frac{\pi}{3}$
      C. $\frac{\pi}{6}$
      D. $\frac{\pi}{2}$

(b) If $u = (3, 1, 4, -6)$ and $v = (-3, -1, -4, 6)$ then the distance between u and v is

A. 0

B. 15

C. $\sqrt{248}$

D. None of the above

(c) Which of the following set of vectors in $\mathbb{R}^3$ is linearly independent?

A. $\{(1, 2, -4), (-8, 14, 6), (3, 4, -9), (1, 0, 0)\}$

B. $\{(1, 2, 5), (2, 5, 1), (1, 5, 2)\}$

C. $\{(1, 2, 3), (0, 0, 0), (3, 2, 1)\}$

D. $\{(3, 2, -4), (24, 16, -32)\}$

(d) The dimension of the vector space of 3×3 matrices of real numbers under the usual addition and scalar multiplication of matrices is

A. infinite

B. 9

C. 6

D. 27

(e) For which value of a and b the vector $w = (1, -3, 4)$ is a linear combination of $u = (2, 4, 0)$ and $v = (1, 4, -2)$ *i.e.* $w = au + bv$?

A. $a = 1, \quad b = -2$

B. $a = -3, \quad b = -2$

C. $a = -1, \quad b = -2$

D. None of the above

(f) If the rank of a 4×4 matrix is equal to 3, then

A. the matrix is invertible.

B. the dimension of the null space is 4.

C. the dimension of the null space is 3.

D. the dimension of the row space is 3.

3. Find a vector that is orthogonal to both $u = (2, -3, 1)$ and $v = (0, 5, 7)$ using cross product. [3]

Solution: We know that the vector $u \times v$ is a vector that is orthogonal to both u and v. Therefore

$$u \times v = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 5 & 7 \end{vmatrix} = -26i - 14j + 10k$$

Hence (-26,-14,10) is the required vector.

4. If $u = (1, -2, 2),\ v = (-3, 0, 4)$ and $w = (6, -3, -2)$, then find $4\|u\| - \|v\| + \|2w\|$. [3]

Solution: Given that

$$\begin{aligned} u &= (1, -2, 2) \Rightarrow \|u\| = 3 \\ v &= (-3, 0, 4) \Rightarrow \|v\| = 5 \\ w &= (6, -3, -2) \Rightarrow 2w = (12, -6, -4) \quad \Rightarrow \|2w\| = 14 \\ Therefore \ \ 4\|u\| - \|v\| + \|2w\| &= 12 - 5 + 14 = 21. \end{aligned}$$

5. Show that the set of vectors $\{(1, 2, 3), (2, 3, 1), (3, 2, 1)\}$, in $\mathbb{R}^3$, is linearly independent. [3]

Solution: We know that the homogeneous system $AX = 0$ have trivial solution if and only if $|A| \neq 0$. In view of this result, if we make the matrix of coefficient of the given vectors and find its determinant. If it is nonzero then we are done. Hence consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}. \text{ Its determinant is } |A| = -12 \neq 0.$$

Therefore the set of vectors $\{(1, 2, 3), (2, 3, 1), (3, 2, 1)\}$, in $\mathbb{R}^3$, is linearly independent.

6. Find the coordinate vector of $w = (7, 5)$ relative to the basis $\{u_1 = (2, -4),\ u_2 = (3, 8)\}$ for $\mathbb{R}^2$. [3]

Solution: We know that each vector of a vector space can be written as the linear combination of the elements of basis. Therefore

$$\begin{aligned} w &= c_1u_1 + c_2u_2 \\ (7, 5) &= c_1(2, -4) + c_2(3, 8) \\ (7, 5) &= (2c_1 + 3c_2, -4c_1 + 8c_2), \quad \text{which gives} \\ 2c_1 + 3c_2 &= 7 \\ -4c_1 + 8c_2 &= 5 \end{aligned}$$

On solving, we get

$$c_1 = \frac{41}{28}, \quad c_2 = \frac{19}{14}.$$

which are the required coordinate vector.

7. Find the dimension of the row space of $A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 3 & 1 \\ 4 & 5 & 5 \end{bmatrix}$. [3]

Solution: We know that the number of non-zero rows of a matrix in echelon form gives the dimension of row space of A. Therefore reduce the given matrix in echelon form, we get

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence, dimension of row space=2.

8. If $T(x_1, x_2, x_3) = (x_1 + x_2, x_1 - x_2 + 3x_3)$, then find [3]

   (i) the domain of T.

   (ii) the codomain of T.

   (iii) the image of (3,4,2).

Solution:

(i) the domain of T is $\mathbb{R}^3$.

(ii) the codomain of T is $\mathbb{R}^2$.

(iii) the image of (3,4,2) is $T(3, 4, 2) = (7, 5)$.