Ministry of Higher Education
Kingdom of Saudi Arabia

College of Science and
Theoretical Studies
Saudi Electronic University

Mid Term Examination Cover Sheet

Mid Term Examination 1436-1437/2015-2016

Course Instructor:		**Exam Date:**	$8^{th} March, 2016$
Course Title:	Linear Algebra	**Course Code:**	Math 251
Exam Duration:	1 Hour	**No. of Pages**	5
CRN:		**Branch:**	

Exam Instructions:

1 Mobile phones are strictly prohibited.

2 Calculators are permitted **WITHOUT** sharing them.

Student Name	
Id Number	

Marking Scheme

Question	Points	Score
1	5	
2	5	
3	3	
4	3	
5	3	
6	3	
7	3	
Total:	25	

1. State whether the following statements are true or false: [5]

(a) The matrix $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ is the inverted coefficient matrix of the following system of linear equation:

$$\begin{aligned} 3x + y &= 4 \\ 5x + 2y &= 7. \end{aligned}$$

(a) True

(b) The determinant of the matrix $\begin{bmatrix} 2 & 6 & 1 \\ 3 & -6 & 9 \\ 0 & 1 & 5 \end{bmatrix}$ is 165.

(b) False

(c) If $u = (7, 1, 5)$ and $v = (5, 4, -1)$ then the distance between u and v is $\sqrt{29}$.

(c) False

(d) Any plane passing through origin is a subspace in $\mathbb{R}^3$.

(d) True

(e) If W is a proper subspace of a finite dimensional vector space V, then $dim(V) \leq dim(W)$.

(e) False

2. Mark Tick (✓) one of the alternatives from the following questions as your answer. [5]

(a) For the matrix $\begin{bmatrix} 3 & 4 & 0 \\ -1 & 2 & 7 \\ -2 & -4 & 4 \end{bmatrix}$, M_{23} is

A. 0
B. -20
C. 4
D. -4

(b) The vector $(1, 2, 0, -1)$ is orthogonal to the vector

A. (-1,3,1,4)
B. (-2,1,0,4)
C. (-2,3,1,4)
D. (-1,3,-1,2)

(c) Which of the following set of vectors in $\mathbb{R}^3$ is a basis?

A. $\{(1, 2, 1), (2, 4, 2), (3, 4, -9)\}$
B. $\{(1, 2, 1), (2, 9, 0), (3, 3, 4)\}$
C. $\{(1, 2, 3), (0, 0, 0), (3, 2, 1)\}$
D. $\{(3, 2, -4), (24, 16, -32)\}$

(d) If θ is the angle between $u = (1, -3, 4)$ and $v = (3, 4, 7)$, then $\cos\theta =$

A. $\frac{19}{26\sqrt{74}}$
B. $\frac{19}{\sqrt{26}\sqrt{74}}$
C. $\frac{19}{\sqrt{26}\sqrt{64}}$
D. $\frac{19}{\sqrt{26}\sqrt{84}}$

(e) If A is a 4×6 matrix with rank 3, then nullity of A is

A. 1
B. 2
C. 4
D. 3

Attempt all questions.

3. Determine whether the following system has no solution, exactly one solution, or infinitely many solutions. [3]

$$\begin{aligned} 2x_1 + 2x_2 &= 2 \\ x_1 + x_2 &= 4 \end{aligned}$$

Solution: No solution.

4. Find the determinant of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$ by cofactor expansion along the first column of A. [3]

Solution: The cofactor expansion along the first column of A is given by

$$\begin{aligned} |A| &= a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31} \\ &= 1(93) + (-4)(-42) + 7(-3) \\ &= 240. \end{aligned}$$

5. Find the cross product $u \times v$ of the vectors $u = (3, 1, 6),\ v = (-2, 5, 7)$. [3]

Solution: $u \times v = \begin{vmatrix} i & j & k \\ 3 & 1 & 6 \\ -2 & 5 & 7 \end{vmatrix} = -23i - 33j + 17k.$

6. Show that the set $S = \{i = (1, 0, 0), j = (0, 1, 0), k = (0, 0, 1)\}$ spans $\mathbb{R}^3$. [3]

Solution: S spans $\mathbb{R}^3$:
Take any element $(x, y, z) \in \mathbb{R}^3$.
This can be written as
$(x, y, z) = x(1, 0, 0) + y(1, 0, 0) + z(1, 0, 0)$
i.e. Every element of $\mathbb{R}^3$ can be written as the linear combination of $(1, 0, 0), (0, 1, 0)$ and $(0, 0, 1)$.
Therefore S spans $\mathbb{R}^3$.

7. Find the coordinate vector of $w = (3, 4)$ relative to the basis $\{u_1 = (1, 0),\ u_2 = (0, 2)\}$ for $\mathbb{R}^2$. [3]

Solution: We know that each vector of a vector space can be written as the linear combination of the elements of basis. Therefore

$$
\begin{aligned}
w &= c_1 u_1 + c_2 u_2 \\
(3, 4) &= c_1(1, 0) + c_2(0, 2) \\
(3, 4) &= (c_1, 2c_2), \quad \text{which gives} \\
c_1 = 3, &\quad c_2 = 2
\end{aligned}
$$

which are the required coordinate vector.