Kingdom of Saudi Arabia
Ministry of Higher Education
Saudi Electronic University

المملكة العربية السعودية
وزارة التعليم العالي
الجامعة السعودية الالكترونية

DISCRETE MATHEMATICS (MATH150)
Level III
ASSIGNMENT-1
2016

Section-I

1.State whether the following statements are True or False.
(9X1=9 Marks)

(a) The conditional statement p → q is true if p is false and q is false.

(a) **True**

(b) The contrapositive of p → q is ¬ q → ¬ p

(b) **True**

(c) Two propositions are logically **e**quivalent if they always have the different truth values for all possible cases of the two statements.

(c) **False**

(d) If *P(x)* denotes "$x < 0$" and domain is set of all integers, then $\forall x\, P(x)$ is false.

(d) **True**

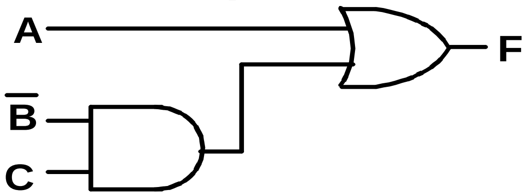
(e) Let $P(x, y)$ denote "$x + y = 0$". The truth value of the quantification $\exists x \forall y P(x, y)$ is true.

(e) **False**

(f) $\neg\forall x\left(P(x) \rightarrow Q(x)\right) \equiv \exists x\left(P(x) \wedge \neg Q(x)\right)$

(f) **True**

(g) For the given logical circuit, the output is F = A + ($\overline{B}$.C) + BC



(g) **False**

(h) In Boolean algebra x + y = y + x

(h) **True**

(i) The value of Boolean expression is $\overline{1.0} + \overline{1 + 0} = 1$.

(i) **True**

Section-II

2. Select one of the alternatives from the following questions as your answer.

(9x1=9 Marks)

(a) How many rows appear in a truth table with three propositions p, q and r?

(A) 3

(B) 4

(C) 6

(D) 8

(b) Negation of $p \wedge q$ is

(A) $\neg p \wedge q$

(B) $\neg p \vee q$

(C) $\neg p \vee \neg q$

(D) $\neg p \wedge \neg q$

(c) The compound proposition $p \wedge (p \vee q)$ is logically equivalent to

(A) p

(B) q

(C) $p \wedge q$

(D) $p \vee q$

(d) The universal quantification $\forall x P(x)$ is false if

(A) $P(x)$ is false for every x

(B) There is an x for which $P(x)$ is false

(C) $P(x)$ is true for every x

(D) $P(x)$ is neither true nor false for every x

(e) Proof by Contrapositive (Indirect proof) of the statement $p \rightarrow q$ is same as proving

(A) $\neg p \rightarrow \neg q$

(B) $\neg p \leftrightarrow \neg q$

(C) $\neg q \rightarrow \neg p$

(D) $q \rightarrow p$

(f) The sum of two positive integer is always positive. Its logical translation is

(A) $\forall x \, \exists y \, ((x > 0) \rightarrow (x + y > 0))$

(B) $\forall x \, \exists y \, ((x > 0) \wedge (y > 0) \rightarrow (x + y > 0))$

(C) $\forall x \, \forall y \, ((x > 0) \wedge (y > 0) \rightarrow (x + y > 0))$

(D) None of the above.

(g) The basic logic gate whose output is the complement of the input is the
- (A) AND Gate
- (B) OR Gate
- (C) Inverter Gate
- (D) NAND Gate

(h) The dual of $x.\bar{z} + x.0 + \bar{x}.1 + (\bar{y}+z)$ is equal to
- (A) $(x+z).(x+1).(\bar{x}+1).(y.\bar{z})$
- (B) $(x+\bar{z}).(x+1).(\bar{x}+0).(\bar{y}.z)$
- (C) $(x+\bar{z}).(x+1).(x+1).(y.z)$
- (D) $(x+z).(x+0).(x+1).(\bar{y}.z)$

(i) The value of $\overline{\bar{x}+\bar{y}} + \overline{\bar{x}.\bar{y}}$ is equal to
- (A) 1
- (B) 0
- (C) xy + x + y
- (D) $x + y + \overline{x.y}$

Section-III

Answer the following Questions **(6 × 2 marks = 12 marks)**

Q 3. By means of a truth table, show that the statement p v ¬ (p Λ q) is a tautology.
Solution: The Truth table for the given statement is:

p	q	p Λ q	¬ (p Λ q)	p v ¬ (p Λ q)
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Since the given statement has truth-values T for all its entries in the last column, the given statement is a tautology.

Q 4. Find the converse and inverse statements of the statement "If you do your homework, then you will be allowed to play".
Solution: Let p(x): You do your homework.
q(x): You will be allowed to play.

Converse for p →q is **q →p**. " If you will be allowed to play, then you do your homework".
Inverse for p →q is ¬ ***p*** → ¬ ***q***. "If you do not do your homework, then you will not be allowed to play".

Q 5. Prove that for an integer *n*, if n^2 is odd, then *n* is odd.
Solution: Proof by method of contraposition.
Proving the statement $p \rightarrow q$ is same as proving $\neg q \rightarrow \neg p$

Assume that n is even.
Therefore, there exists an integer k such that n = 2k .
Now $n^2 = 4k^2 = 2(2k^2)$
Which implies that n^2 is even.
We have shown that if n is an even integer, then n^2 is also even.
Therefore by contraposition, for an integer n, if n^2 is odd, then n is odd.

Q 6. Over the universe of animals, let
p(x): x is a whale;
q(x): x is a fish;
r(x): x lives in water.
Write the following sentences as quantified statements:
(1) There exists an animal, which does not live in water.
(2) There exists a fish that is not a whale.
(3) Every whale that lives in the water is a fish.
Solution: (1) ∃ x (¬ r(x)).
(2) ∃ x (q(x) Λ ¬ p(x)).
(3) ∀ x ((p(x) Λ r(x) → q(x)).

Q 7. Verify the distributive law by constructing a truth table for the Boolean variables x, y, z
x+ y. z = (x + y). (x +z)

Solution:

x	y	z	y.z	x+yz	x+y	x+z	(x+y)(x+z)
1	1	1	1	**1**	1	1	**1**
1	1	0	0	**1**	1	1	**1**
1	0	1	0	**1**	1	1	**1**
1	0	0	0	**1**	1	1	**1**
0	1	1	1	**1**	1	1	**1**
0	1	0	0	**0**	1	0	**0**
0	0	1	0	**0**	0	0	**0**
0	0	0	0	**0**	0	0	**0**

We can see from the truth table, column 5 and column 8 are identical

Q 8. Write f(x, y, z)= (x + y.z).($\bar{x}$ + y.z) as sum of products for the Boolean variables x, y, z.
Solution:

x	y	z	$\bar{x}$	y.z	x+ y.z	$\bar{x}$ +y.z	(x+ y.z).($\bar{x}$ +y.z)
1	1	1	0	1	1	1	1
1	1	0	0	0	1	0	0
1	0	1	0	0	1	0	0
1	0	0	0	0	1	0	0
0	1	1	1	1	1	1	1
0	1	0	1	0	0	1	0
0	0	1	1	0	0	1	0
0	0	0	1	0	0	1	0

In the last column only two 1's are present, so we have to write two terms related to input variables x, y and z. Therefore, **f(x, y, z) = x.y.z+ $\bar{x}$ .y. z**