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MATH 150

Lecture Notes in a Capsule

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SAUDI ELECTRONIC UNIVERSITY
RIYADH

Table of Contents

$\boldsymbol{Week\ 7: Proof\ by\ Induction}$

$Steps:$

$1 - \boldsymbol{Basis\ Step}: Show\ p(1)\ is\ true.$

$2 - \boldsymbol{Inductive\ Step}: Show\ for\ all\ Z^{+}(postive\ integers), if\ p(k)\ true, then\ p(k+1)\ is\ true.$

$\boldsymbol{Example\ 1}: use\ mathimatical\ induction\ to\ show$ $1+2+3+\cdots.+n=\frac{n(n+1)}{2}$

$\boldsymbol{Sloution}:$

$\boldsymbol{1 - Basis\ Step} : p(1)\ is\ true\ since: 1=\frac{1(1+1)}{2}=1\ .$

$Therefore, \boldsymbol{Left\ Hand\ Side}\ (\boldsymbol{LHS}) = \boldsymbol{Right\ Hand\ Side}\ (\boldsymbol{RHS})$

$\boldsymbol{2 - Inductive\ Step} : Assume\ \ that\ p(k) is\ true\ i.e.\ 1+2+3+\cdots+k=\frac{k(k+1)}{2}$

$by\ showing\ that\ p(k+1)\ is\ ture,$

$that\ is, p(k+1) = 1+2+3+\cdots+\boldsymbol{k}+(k+1)=\frac{k+1\big((k+1)+1\big)}{2}=\frac{(k+1)(k+2)}{2}$

$We\ need\ to\ arrive\ to\ this$

$Strating\ from:\ p(k+1) =.1+2+3+\cdots+\boldsymbol{k}+(k+1)=\frac{k(k+1)}{2}+(k+1)=$

$$\frac{[k(k+1)+\ 2(k+1)]}{2}=\frac{[(k^2+k+2k+2)]}{2}=\frac{(k+1)(k+2)}{2}\ \therefore p(k+1) is\ true\ ✓$$

Self-Check Question : Fill up the Red gaps (Check Last Page for Full Answers)

$\boldsymbol{Example\ 1}$: $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$\boldsymbol{Sloution}$:

$\boldsymbol{Basis\ Step}$: $p(1)$ *is true since*: $1^2 = \frac{1(1+1)(2(1)+1}{6} = \frac{(2)(3)}{6} = \frac{6}{6} = 1$. *Therefore* $LHS = RHS$

$\boldsymbol{Inductive\ Step}$: *Assume that* $p(k)$ *is true i.e.* $1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{????????}{6}$

by showing that $p(k+1)$ *is ture, that is,*

$$p(k+1) = 1^2 + 2^2 + 3^2 + \cdots + (k+1)^2 = \frac{(k+1)\big((k+1)+1\big)(2(k+1)+1)}{6}$$

$$= \frac{?????????}{6}\ We\ need\ to\ arrive\ to\ this$$

Strating from: $p(k+1) = .1 + 2 + 3 + \cdots + \boldsymbol{k^2} + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$

$$\frac{k(k+1)(2k+1) + ??????}{6} = \frac{(k+1)[k(2k+1)+6(k+1)]}{6} = \frac{(k+1)[?????????]}{6}$$

$$= \frac{(k+1)[??+??+4k+6]}{6} = \frac{(k+1)[(k+2)(2k+3)]}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$\therefore p(k+1)$ *is true* ✓

Note: Easy way to arrive to the equation is to break down $(k+2)(2k+3)$ and find how to re-group it.

$\boldsymbol{Example\ 2}$: $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

$\boldsymbol{Sloution}$: $\boldsymbol{Basis\ Step}$: $p(1)$ *is true since*: $1^3 = ????????$. *Therefore* $???? = ????$

$\boldsymbol{Inductive\ Step}$: *Assume that* $p(k)$ *is true i.e.* $1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}$

by showing that $p(k+1)$ *is ture, that is,*

$$p(k+1) = 1^3 + 2^3 + 3^3 + \cdots + k^3 + ???? = \frac{??????}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4}\ We\ need\ to\ arrive\ to\ this$$

Strating from: $p(k+1) = 1^3 + 2^3 + 3^3 + \cdots + k^3 + ???? = \frac{k^2(k+1)^2}{4} + ???? =$

$$\frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2[k^2+4(k+1)]}{4} = \frac{(k+1)^2[k^2+4k+4]}{4} = \frac{(k+1)^2\ ??????}{4}$$

$\therefore p(k+1)$ *is true*

$Week\ 7$: $Recursive\ Defintion$ & $Structured\ Induction$

$Steps$:

$1 -$ $\boldsymbol{Basis\ Step}$: $Starting\ value\ of\ a_1$.

$2 -$ $\boldsymbol{Recursive\ Step}$: $The\ recursive\ equation\ for\ (an)\ as\ (a)\ is\ a\ function\ of\ an_{-1}$.

$\boldsymbol{Example\ 1}$: $suppose\ f\ is\ defined\ recursivly$: $f_{(0)} = 3$, $where\ f_{(n+1)} = 2f_{(n)} + 3$.

$$Find: f(1), f(2), f(3), and\ f(4)?$$

$\boldsymbol{Sloution}$: $we\ have$:

$$f_{(n+1)} = 2f_{(n)} + 3, and\ f_{(0)} = 3$$

$$so\ f_{(1)} = 2f_{(0)} + 3 = 2(3) + 3 = 9$$

$$f_{(2)} = 2f_{(1)} + 3 = 2(9) + 3 = 21$$

$$f_{(3)} = 2f_{(2)} + 3 = 2(21) + 3 = 45$$

$$f_{(4)} = 2f_{(3)} + 3 = 2(45) + 3 = 93$$

Self-Check Question : Fill up the Red gaps (Check Last Page for Full Answers)

$\boldsymbol{Example\ 1}$: $suppose\ f\ is\ defined\ recursivly$: $a_1 = -4$, $where\ an = an_{-1} + 5$

$$Find: a_2, a_3, a_{4,}\ and\ a_5?$$

$\boldsymbol{Sloution}$: $we\ have$:

$$a_n = a_{n-1} + 5, and\ a_1 = -4$$

$$so, a_2 = a_{2-1} + 5 = a_1 + 5 = ?? + 5 = 1$$

$$a_3 = a_{3-1} + 5 = a_2 + 5 = ?? + 5 = ??$$

$$a_4 = a_{4-1} + 5 = ?? + 5 = ?? + 5 = 11$$

$$a_5 = a_{5-1} + 5 = a_4 + 5 = 11 + 5 = 16$$

$\boldsymbol{Week\ 8: Counting}$

$Two\ Rules\ of\ Counting:$

$1 - \boldsymbol{Product\ Rule}: where\ we\ multiply\ the\ given\ choices: n_1 x\ n_2 = answer.$

$2 - \boldsymbol{Sum\ Rule}: where\ we\ multiply\ the\ given\ choices: n_1 +\ n_2 = answer.$

$\boldsymbol{Example\ 1}: In\ a\ company\ there\ are\ \mathbf{2}\ employees: Ali\ and\ Mohammad, and\ \mathbf{12}\ offices.$

$How\ mant\ different\ ways\ to\ assign\ different\ offices\ to\ each\ employee?$

$\boldsymbol{Solution}: Ali\ have\ \mathbf{12}\ choices.$ $\boldsymbol{After\ he\ chose}, Mohammad\ will\ have\ \mathbf{11}\ choices.$

$Therefore, there\ will\ be\ \mathbf{11}(\mathbf{12}) = \mathbf{132}\ ways\ to\ assign\ offices\ to\ each\ employee.$

$\boldsymbol{Example\ 2}:$

$Suppose\ there\ is\ a\ big\ hall\ and\ \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C} \ldots \boldsymbol{Z}\ chairs, where\ each\ chair\ has\ \mathbf{100}\ ways\ be\ leveled.$

$How\ many\ ways\ to\ level\ all\ chairs?$

$\boldsymbol{Solution}: since\ from\ A - Z\ is\ 26\ chairs, and\ 100\ ways\ for\ each\ chair,$

$so\ there\ are\ \mathbf{26}(\mathbf{100}) = \mathbf{2600}\ ways\ to\ level\ the\ chairs.$

Self-Check Question : Fill up the Red gaps (Check Last Page for Full Answers)

$\boldsymbol{Example\ 1}:\ Among\ any\ group\ of\ 366\ people, there\ must\ be\ at\ least\ 2\ with\ the\ same\ birthday.$

$Yes\ or\ no? Justify?$

$This\ is\ true, according\ to\ \boldsymbol{pigeonhole\ principle}, and$

$scince\ we\ have\ ???\ days\ in\ a\ year, and\ 366\ people. So\ there\ must\ be\ 2\ with$

$the\ same\ birth\ day.$

$\boldsymbol{Week\ 8: Permutation\ \&\ Combinations}$

$Defintions:$

$1 - \boldsymbol{Permutation}$: *is an arrangement of objects in a* **specific order**.

$$Formula: \boldsymbol{nPr} = \frac{\boldsymbol{n}!}{(\boldsymbol{n}-\boldsymbol{r})!}$$

$2 - \boldsymbol{Combinations}$: *is an arrangement of objects* **without order**.

$$Formula: \boldsymbol{nCr} = \binom{\boldsymbol{n}}{\boldsymbol{r}} = \frac{\boldsymbol{n}!}{\boldsymbol{r}!\ (\boldsymbol{n}-\boldsymbol{r})!}$$

$$Note: n\boldsymbol{C}0 = n\boldsymbol{C}n = 1; n\boldsymbol{C}1 = n\boldsymbol{C}(n-1) = n \quad Also, n\boldsymbol{Cr} = n\boldsymbol{C}(n-r)$$

Permutation Example 1 : *Compute* **6P2** ?

$$\boldsymbol{Slloution}: \boldsymbol{6P2} = \frac{\boldsymbol{6}!}{(\boldsymbol{6}-\boldsymbol{2})!} = \frac{\boldsymbol{6}!}{\boldsymbol{4}!} = \frac{\boldsymbol{6.5.}\cancel{\boldsymbol{4.3.2.1}}}{\cancel{\boldsymbol{4.3.2.1}}} = \boldsymbol{6.5} = \boldsymbol{30}$$

Permutation Example 2: *Find the number of ways to* **arrange** **5 objects** *from* **7 different objects**?

$$\boldsymbol{Slloution}: \boldsymbol{7P5} = \frac{\boldsymbol{7}!}{(\boldsymbol{7}-\boldsymbol{5})!} = \frac{\boldsymbol{7}!}{\boldsymbol{2}!} = \frac{\boldsymbol{7.6.5.4.3.}\cancel{\boldsymbol{2.1}}}{\cancel{\boldsymbol{2.1}}} = \boldsymbol{7.6.5.4.3} = \boldsymbol{60}$$

Combinatoin Example 1 : *Compute* **7C2** ?

$$\boldsymbol{Slloution}: \boldsymbol{7C2} = \binom{\boldsymbol{7}}{\boldsymbol{2}} = \frac{\boldsymbol{7}!}{\boldsymbol{2}!\,(\boldsymbol{7}-\boldsymbol{2})!} = \frac{\boldsymbol{7}!}{\boldsymbol{2}!\,(\boldsymbol{5})!} = \frac{\boldsymbol{7.6.}\cancel{\boldsymbol{5.4.3.2.1}}}{\boldsymbol{2.1}(\cancel{\boldsymbol{5.4.3.2.1}})} = \frac{\boldsymbol{7.6}}{\boldsymbol{2}} = \frac{\boldsymbol{42}}{\boldsymbol{2}} = \boldsymbol{21}$$

Combination Example 2: *There are* **5 people** *in a club,* **3 of those** *are going to be in a planinig committee. How many ways this committe is created?*

$$\boldsymbol{Slloution}: \boldsymbol{5C3} = \binom{\boldsymbol{5}}{\boldsymbol{3}} = \frac{\boldsymbol{5}!}{\boldsymbol{3}!\,(\boldsymbol{5}-\boldsymbol{3})!} = \frac{\boldsymbol{5}!}{\boldsymbol{3}!\,(\boldsymbol{2})!} = \frac{\boldsymbol{5.4.}\cancel{\boldsymbol{3.2.1}}}{\cancel{\boldsymbol{3.2.1}}(\boldsymbol{2.1})} = \frac{\boldsymbol{5.4}}{\boldsymbol{2}} = \frac{\boldsymbol{20}}{\boldsymbol{2}} = \boldsymbol{10}$$

Note: we used combination since no specific order is specified.

Self-Check Question : Fill up the Red gaps (Check Last Page for Full Answers)

$\boldsymbol{Example\ 1}$: $How\ many\ \boldsymbol{different}\ \mathbf{3}\ digits\ can\ be\ made\ from: \mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}, \mathbf{8}, if\ a\ digit\ can$

$appear\ \boldsymbol{only\ once}?$

$\boldsymbol{Sloution}: \mathbf{5}?\,\mathbf{3} = \frac{?}{??} = ?\,\boldsymbol{x}?\,\boldsymbol{x}? = \mathbf{60}$

$\boldsymbol{Example\ 2}$: $How\ \boldsymbol{many\ ways}\ can\ a\ coach\ choose\ \mathbf{3}\ swimmers\ from\ among\ \mathbf{5}\ \boldsymbol{swimmers}?$

$\boldsymbol{Sloution}: \mathbf{5}?\,\mathbf{3} = \frac{?}{??} = ?\,\boldsymbol{x}?\,\boldsymbol{x}? = \mathbf{10}$

Week 9: Binomial Theorem

Defintion:

Binomial Theorem*: is a polynomial with two terms Ex*: $5y^3 - 3$

Now what happen if we multiply binomial by itself many times:

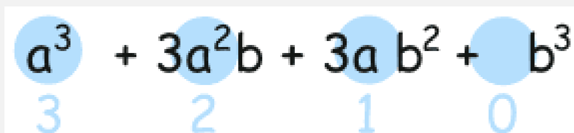
$(\boldsymbol{a}+\boldsymbol{b})^{\mathbf{0}} = \mathbf{1}$

$(\boldsymbol{a}+\boldsymbol{b})^{\mathbf{1}} = (\boldsymbol{a}+\boldsymbol{b})$

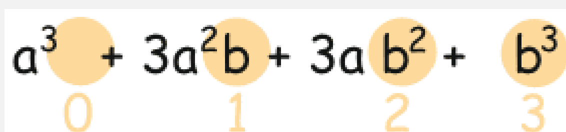
$(\boldsymbol{a}+\boldsymbol{b})^{\mathbf{2}} = \boldsymbol{a}^{\mathbf{2}} + \mathbf{2}\boldsymbol{ab} + \boldsymbol{b}^{\mathbf{2}}$

$(\boldsymbol{a}+\boldsymbol{b})^{\mathbf{3}} = \boldsymbol{a}^{\mathbf{3}} + \mathbf{3}\boldsymbol{a}^{\mathbf{2}}\boldsymbol{b} + \mathbf{3}\boldsymbol{ab}^{\mathbf{2}} + \boldsymbol{b}^{\mathbf{3}}$

Now, notice the exponents of ***a****. They start at* 3 *and go down*: 3, 2, 1, 0:



Likewise, the exponents of ***b*** *go upwards*: 0, 1, 2, 3:



Therefore, we can conclude:

$$
\begin{aligned}
(a+b)^3 &= \sum_{k=0}^{3} \binom{3}{k} a^{3-k} b^k \\
&= \binom{3}{0} a^{3-0} b^0 + \binom{3}{1} a^{3-1} b^1 + \binom{3}{2} a^{3-2} b^2 + \binom{3}{3} a^{3-3} b^3 \\
&= 1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3 \cdot a^1 b^2 + 1 \cdot a^0 b^3 \\
&= a^3 + 3a^2 b + 3ab^2 + b^3
\end{aligned}
$$

For more good details:

http://www.mathsisfun.com/algebra/binomial-theorem.html

$Example\ 1: compute\ \binom{6}{4}\ ?$

$Slountion: \binom{6}{4} = \frac{6!}{4!\,(6-4)!} = \frac{6!}{4!\,(2)!} = \frac{6.5.\cancel{4.3.2.1}}{\cancel{4.3.2.1}(2.1)} = \frac{30}{2} = 15$

$Example\ 2: find\ (a+b)^6\ using\ the\ Binomial\ Theorem?$

$$Slountion: (a+b)^6 = \binom{6}{0}a^6 + \binom{6}{1}a^5b + \binom{6}{2}a^4b^2 + \binom{6}{3}a^3b^3 + \binom{6}{4}a^2b^4 + \binom{6}{5}ab^5 + \binom{6}{6}b^6$$

$$= a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$$

Self-Check Question : Fill up the Red gaps (Check Last Page for Full Answers)

$Example\ 1: compute\ \binom{7}{4}\ ?$

$Slountion: \binom{7}{4} = \frac{7!}{?!\,(?-?)!} = \frac{7!}{?!\,(3)!} = \frac{7.6.5.\cancel{4.3.2.1}}{\cancel{??????}(3.2.1)} = \frac{210}{6} = ??$

$Example\ 2: find\ (a+b)^7\ using\ the\ Binomial\ Theorem?$

$Slountion:$

$$(a+b)^7 = \binom{7}{0}a^? + \binom{7}{1}a^?b + \binom{7}{2}a^?b^2 + \binom{7}{3}a^?b^3 + \binom{7}{4}a^?b^4 + \binom{7}{5}a^?b^5 + \binom{7}{6}a^?b^6 + \binom{7}{7}b^7$$

$$=??$$

Week 9: Advance Counting Techniques

Defintion:

Sequence: *is a discrete structure used to represent an ordered list.*

For Example: 1,3,5,7,11 *are sequence terms or intial sequence.*

Example 1: $\{a_n\}$ *is a sequnce satisfying* $\boldsymbol{a_n = a_{n-1} - a_{n-2}}$; ***where*** $\boldsymbol{a_0 = 3}$ ***and*** $\boldsymbol{a_1 = 5}$.

Find $\boldsymbol{a_2}$ *and* $\boldsymbol{a_3}$?

Sloution:

$$Since\ \boldsymbol{a_n = a_{n-1} - a_{n-2}}$$

$$,\boldsymbol{a2 = a_{2-1} - a_{2-2}}$$

$$\boldsymbol{a2 = a_1 - a_0}$$

$$\boldsymbol{a2 = 5 - 3 = 2},$$

$$Therefore, \boldsymbol{a_2 = 2}$$

$$Similarly, \boldsymbol{a_3 = a_{3-1} - a_{3-2}}$$

$$\boldsymbol{a_3 = a_2 - a_1}$$

$$\boldsymbol{a_3 = 2 - 5 = -3}$$

$$Therefore, \boldsymbol{a_3 = -3}$$

Self-Check Question : Fill up the Red gaps (Check Last Page for Full Answers)

Example 1: $\{a_n\}$ *is a sequnce satisfying* $\boldsymbol{a_n = a_{n-1} + a_{n-2}}$; ***where*** $\boldsymbol{a_0 = 1}$ ***and*** $\boldsymbol{a_1 = 2}$.

Find $\boldsymbol{a_2, a_3}$ ***and*** $\boldsymbol{a_4}$?

$$Since\ \boldsymbol{a_n = a_{n-1} + a_{n-2}}$$

$$,\boldsymbol{a2 = a_{2-1} + a_{2-2}}$$

$$\boldsymbol{a2} = ?? + ??$$

$$\boldsymbol{a2} = ?? + ?? = \boldsymbol{3}$$

$$Similarly, \boldsymbol{a_3 = a_{3-1} + a_{3-2}}$$

$$\boldsymbol{a_3 = a_2 + a_1}$$

$$\boldsymbol{a_3} = ?? + ?? = ??$$

$$Likewise, \boldsymbol{a_4 = a_{4-1} + a_{4-2}} = ?? + ?? = ??$$

$\textbf{Week 10: Solving Recurrence Relation}$

$Therome: given\ \boldsymbol{a_k} = \boldsymbol{A_{ak_{-1}}} + \boldsymbol{B_{ak_{-2}}}\ if\ \boldsymbol{S}, \boldsymbol{T}, \boldsymbol{C}, \boldsymbol{D}\ are\ nonzero\ real\ numbers\ then\ its$

$\boldsymbol{general\ sluotion}\ is\ \boldsymbol{a_k} = \boldsymbol{C}(\boldsymbol{S^k}) + \boldsymbol{D}(\boldsymbol{T^k})$.

$$\boldsymbol{and}, the\ characterstic\ equation\ of\ the\ relation\ \boldsymbol{x^2} = \boldsymbol{Ax} + \boldsymbol{B}$$

$$\boldsymbol{Also}, the\ characterstic\ polynomial\ of\ the\ relation\ \boldsymbol{x^2} - \boldsymbol{Ax} - \boldsymbol{B}$$

$\boldsymbol{Example\ 1}: let\ \boldsymbol{a_k} = \boldsymbol{5_{ak_{-1}}} - \boldsymbol{6_{ak_{-2}}}\ find\ the\ general\ sluotion?$

$\boldsymbol{Sloution}:$

$\boldsymbol{The\ characteristic\ equation\ is}:$

$$x^2 - 5x - (-6) = 0$$

$$x^2 - 5x + 6 = 0$$

$$(x - 2)(x - 3) = 0$$

$$\therefore x = 2\ , 3$$

$\boldsymbol{The\ general\ sluotion\ is}:$

$$a_n = C(2^n) + D(3^n)$$

$\boldsymbol{Example\ 2}: find\ the\ \boldsymbol{particular\ sloution}\ of\ \boldsymbol{Example\ 1\ when}\ \boldsymbol{a_0 = 9; a_1 = 20}$?

$\boldsymbol{Sloution}:$

$$Now\ since\ 7 + 2 = 20, a_0 = C(2^0) + D(3^0) = C + D = 9$$

$$and\ since\ 14 + 6 = 20, a_1 = C(2^1) + D(3^1) = 2C + 3D = 20$$

$$Now\ lets\ multiply\ a_0\ by\ 2. so,\ \ (2)a_0 = 2C + 2D = (2)9 = 18$$

$$and\ bring\ \boldsymbol{a_1}\ down, so \qquad - \quad a_1 = 2C + 3D = 20$$

$Now\ substract, you\ will\ get \qquad -D = -2\ or\ \boldsymbol{D = 2}$

$Now\ lets\ plug\ in\ D\ in\ a_0 or\ a_1 to\ get\ C.$

$$If\ we\ use\ a_1, we\ will\ get\ a_1 = 2C + 3(2) = 20$$

$$a_1 = 2C = 20 - 6 \rightarrow a_1 = 2C = 14 \rightarrow a_1 = \boldsymbol{C = 7}$$

$Therefore, thr\ particular\ sloution\ is\ a_n = 7(2^n) + 2(3^n).$

$\boldsymbol{Week\ 11: Relations}$

$Defintion: let\ \boldsymbol{A\ and\ B}\ \ be\ 2\ nonempty\ sets, then\ a\ realtion\ from\ A\ to\ B\ is\ a\ subset\ of\ AXB$

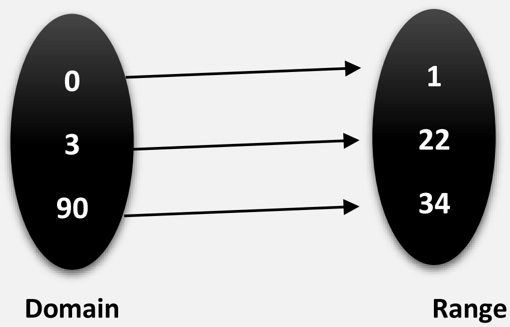
$\boldsymbol{R\ is}\ \ a\ relation\ from\ A \to\ B\ \ \leftrightarrow R \leq AXB, here\ \boldsymbol{A\ is\ called\ Domain\ and\ B\ range}.$

$\boldsymbol{Example\ 1}: let\ \boldsymbol{R} = [(\boldsymbol{0}, \boldsymbol{1}), (\boldsymbol{3}, \boldsymbol{22}), (\boldsymbol{90}, \boldsymbol{34})]\ \ \boldsymbol{find\ the\ Domain\ and\ Range}?$

$\boldsymbol{Sloution}: \boldsymbol{Domain} = \boldsymbol{0}, \boldsymbol{3}, \boldsymbol{and\ 90}; \boldsymbol{Range} = \boldsymbol{1}, \boldsymbol{22}, \boldsymbol{and\ 34}.$

$\boldsymbol{Example\ 2}: \boldsymbol{Represent\ R\ in\ Example\ 1\ by\ Arrow\ Charts}?$

$\boldsymbol{Sloution}:$



$\boldsymbol{Example\ 3}: \boldsymbol{Let\ A} = [\boldsymbol{0}, \boldsymbol{1}, \boldsymbol{2}], \boldsymbol{and\ B} = [\boldsymbol{a}, \boldsymbol{b}]\ \boldsymbol{then}\ (\boldsymbol{0}, \boldsymbol{a}), (\boldsymbol{0}, \boldsymbol{b}), (\boldsymbol{1}, \boldsymbol{a}), (\boldsymbol{2}, \boldsymbol{b})$

$\boldsymbol{is\ a\ relation\ from\ A\ to\ B. Represent\ the\ given\ relation\ by\ Matrix}?$

$\boldsymbol{Sloution}:$

R	a	B
0	X	X
1	X	
2		X

$\boldsymbol{Week\ 11}$: $\boldsymbol{Types\ of Relations}$

$\boldsymbol{a}$) $\boldsymbol{Reflexive\ Relation}$: $is\ said\ to\ be\ reflexsive\ \boldsymbol{if}\ (\boldsymbol{x}, \boldsymbol{x},) \in \boldsymbol{R}\ \forall \boldsymbol{x}\ \in \boldsymbol{R}$

$\boldsymbol{Example\ 1}$:

$The\ relation\ is\ less\ than\ or\ equal\ to\ i.e. \leq is\ the\ set\ of\ natural\ numbers\ and\ is\ Reflexive$

$\boldsymbol{Therefore, n \leq n, since\ 1 \leq 1, 2 \leq 2, 3 \leq 3, \ldots \ldots}$

$\boldsymbol{Example\ 2}$:

$Let\ L1\ and\ L2\ be\ two\ parallel\ lines,$

$then\ L1\ is\ Parallel\ to\ L2,$

$or\ L2\ is\ Parallel\ to\ L1.$

$\boldsymbol{Therefore, the\ relation\ between\ L1\ and\ L2\ is\ Reflexive}.$

$\boldsymbol{b}$) $\boldsymbol{Symetric\ Relation}$: $is\ said\ to\ be\ Symetric\ \boldsymbol{if}\ (\boldsymbol{y}, \boldsymbol{x},) \in \boldsymbol{R}\ \ \boldsymbol{whenever}\ (\boldsymbol{x}, \boldsymbol{y},) \in \boldsymbol{R}$

$\forall\ (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{R}.$

$\boldsymbol{Example\ 1}$:

$Let\ L1\ and\ L2\ be\ two\ perpendicular\ lines,$

$then\ L1\ is\ perpendicular\ to\ L2,$

$or\ L2\ is\ perpendicular\ to\ L1$

$\boldsymbol{Therefore, the\ relation\ between\ L1\ and\ L2\ is\ Symetric}.$

$\boldsymbol{b}$) $\boldsymbol{Transitive\ Relation}$: $is\ said\ to\ be\ Transitive\ \boldsymbol{iff}\ (\boldsymbol{x}, \boldsymbol{y}) \in \boldsymbol{R}\ \ \boldsymbol{and}\ (\boldsymbol{y}, \boldsymbol{z},) \in \boldsymbol{R} \rightarrow (\boldsymbol{x}, \boldsymbol{z},) \in \boldsymbol{R}.$

$\boldsymbol{Example\ 1}$:

$Let\ L1, L2, and\ L3\ be\ three\ parallel\ lines,$

$then\ L1\ is\ Parallel\ to\ L2$

$or\ L2\ is\ Parallel\ to\ L3$

$also, L1\ is\ Parallel\ to\ L3$

$\boldsymbol{Therefore, the\ relation\ between\ L1, L2, and\ L3\ is\ Transitive}.$

$\boldsymbol{Example\ 1: consider\ the\ following\ relation\ on\ [1,2,3,4]\ where:}$

$\boldsymbol{R1} = [(\boldsymbol{1},\boldsymbol{1}),(\boldsymbol{1},\boldsymbol{2}),(\boldsymbol{2},\boldsymbol{1}),(\boldsymbol{2},\boldsymbol{2}),(\boldsymbol{2},\boldsymbol{2}),(\boldsymbol{3},\boldsymbol{4}),(\boldsymbol{4},\boldsymbol{1}),(\boldsymbol{4},\boldsymbol{4})]$

$\boldsymbol{R2} = [(\boldsymbol{1},\boldsymbol{1}),(\boldsymbol{1},\boldsymbol{2}),(\boldsymbol{2},\boldsymbol{1})]$

$\boldsymbol{R3 = [(1,1),(1,2),(1,4),(2,1),(2,2),(3,3),(4,4),(4,1)]}$

$\boldsymbol{R4 = [(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(3,3),(3,4),(4,4)]}$

$\boldsymbol{R5} = [(\boldsymbol{3},\boldsymbol{4})\ ]$

$\boldsymbol{We\ can\ see\ that\ R4\ is\ Reflexive\ to\ R3, since}\ (\boldsymbol{1},\boldsymbol{1}),(\boldsymbol{2},\boldsymbol{2}),(\boldsymbol{3},\boldsymbol{3}),(\boldsymbol{4},\boldsymbol{4})\ \boldsymbol{are\ in\ R3\ and\ R4}$

Full Answers: $\textbf{\textit{Week 7: Proof by Induction}}$

$\textbf{\textit{Example 1}}$: $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

$\textbf{\textit{Basis Step}}$: $p(1)$ *is true since*: $1^2 = \frac{1(1+1)(2(1)+1}{6} = \frac{(2)(3)}{6} = \frac{6}{6} = 1$. *Therefore LHS = RHS*

$\textbf{\textit{Inductive Step}}$: *Assume that* $p(k)$ *is true i.e.* $1^2 + 2^2 + 3^2 + \cdots + k^2 = \frac{k(k+1)(2k+1)}{6}$

by showing that $p(k+1)$ *is ture, that is,*

$$p(k+1) = 1^2 + 2^2 + 3^2 + \cdots + (k+1)^2 = \frac{(k+1)\big((k+1)+1\big)(2(k+1)+1)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6} \quad \textit{We need to arrive to this}$$

Strating from: $p(k+1) = .1 + 2 + 3 + \cdots + \boldsymbol{k^2} + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2 =$

$$\frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \frac{(k+1)[k(2k+1) + 6(k+1)]}{6} = \frac{(k+1)[2k^2 + k + 6k + 6]}{6}$$

$$= \frac{(k+1)[2k^2 + 3k + 4k + 6]}{6} = \frac{(k+1)[(k+2)(2k+3)]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6} \quad \therefore p(k+1) \textit{ is true} \checkmark$$

Note: Easy way to arrive to the equation is to break down $(k+2)(2k+3)$ and find how to re-group it.

$\textbf{\textit{Example 2}}$: $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

$\textbf{\textit{Basis Step}}$: $p(1)$ *is true since*: $1^3 = \frac{1^2(1+1)^2}{4} = \frac{(1)(4)}{4} = \frac{4}{4} = 1$. *Therefore LHS = RHS*

$\textbf{\textit{Inductive Step}}$: *Assume that* $p(k)$ *is true i.e.* $1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}$

by showing that $p(k+1)$ *is ture, that is,*

$$p(k+1) = 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{(k+1)^2\big((k+1)+1\big)^2}{4} = \frac{(k+1)^2(k+2)^2}{4}$$

We need to arrive to this

Strating from: $p(k+1) = 1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3 =$

$$\frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2[k^2 + 4(k+1)]}{4} = \frac{(k+1)^2[k^2 + 4k + 4]}{4} = \frac{(k+1)^2(k+2)^2}{4}$$

$\therefore p(k+1)$ *is true* ✓

Full Answers: ***Week* 7: *Recursive Defintion* & *Structured Induction***

***Example* 1**: *suppose f is defined recursivly*: $a_1 = -4$, *where* $an = an_{-1} + 5$

Find: $a_2, a_3, a_{4,}$ *and* a_5?

Sloution: *we have*:

$$a_n = a_{n-1} + 5, \text{and } a_1 = -4$$

$$so, a_2 = a_{2-1} + 5 = a_1 + 5 = -4 + 5 = 1$$

$$a_3 = a_{3-1} + 5 = a_2 + 5 = 1 + 5 = 6$$

$$a_4 = a_{4-1} + 5 = a_3 + 5 = 6 + 5 = 11$$

$$a_5 = a_{5-1} + 5 = a_4 + 5 = 11 + 5 = 16$$

Full Answers: ***Week* 8: *Counting***

***Example* 1**: *Among any group of* 366 *people, there must be at least* 2 *with the same birthday.*

Yes or no? Justify?

This is true, according to ***pigeonhole principle****, and*

scince we have 365 *days in a year, and* 366 *people. So there must be* 2 *with*

the same birth day.

Full Answers: ***Week* 8: *Permutation* & *Combinations***

Example* 1**: *How many* ***different* 3** *digits can be made from*: **4, 5, 6, 7, 8**, *if a digit can appear* ***only once?

Sloution: $5P3 = \frac{5!}{(5-3)!} = 5x4x3 = 60$

Example* 2**: *How* ***many ways *can a coach choose* **3** *swimmers from among* **5** ***swimmers***?

Sloution: $5C3 = \frac{5!}{3!\,(5-3)!} = \frac{5x4x3}{3x2x1} = 10$

Full Answers: $\textit{Week 9: Binomial Theorem}$

Example 1: compute $\binom{7}{4}$?

SlOution: $\binom{7}{4} = \frac{7!}{4!\,(7-4)!} = \frac{7!}{4!\,(3)!} = \frac{7.6.5.\cancel{4.3.2.1}}{\cancel{4.4.2.1}\,(3.2.1)} = \frac{210}{6} = 35$

Example 2: find $(a+b)^7$ ***using the Binomial Theorem***?

Slotion:

$$(a+b)^7 = \binom{7}{0}a^7 + \binom{7}{1}a^6b + \binom{7}{2}a^5b^2 + \binom{7}{3}a^4b^3 + \binom{7}{4}a^3b^4 + \binom{7}{5}a^2b^5 + \binom{7}{6}a^1b^6 + \binom{7}{7}b^7$$

$$= a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7a^1b^6 + b^7$$

Full Answers: $\textit{Week 9: Advance Counting Techniques}$

Example 1: $\{a_n\}$ *is a sequnce satisfying* $a_n = a_{n-1} + a_{n-2}$; **where** $a_0 = 1$ **and** $a_1 = 2$.

Find a_2, a_3 **and** a_4?

$$\textit{Since } a_n = a_{n-1} + a_{n-2}$$

$$, a2 = a_{2-1} + a_{2-2}$$

$$a2 = a_1 + a_0$$

$$a2 = 2 + 1 = 3$$

$$\textit{Similarly}, a_3 = a_{3-1} + a_{3-2}$$

$$a_3 = a_2 + a_1$$

$$a_3 = 3 + 2 = 5$$

$$\textit{Likewise}, a_4 = a_{4-1} + a_{4-2} = 5 + 3 = 8$$