Chapter – 12 (Summary)
Boolean Algebra
Week – 3

Boolean algebra works with the set $\{0, 1\}$. The operations are

Boolean Sum $+$ or (OR)

$1+1=1,\ 1+0=1,\ 0+1=1,\ 0+0=0$

Boolean Product $\cdot$ or (AND)

$1\cdot 1=1,\ 1\cdot 0=0,\ 0\cdot 1=0,\ 0\cdot 0=0$

Compliment of an element

$\bar{0}=1\ ,\ \bar{1}=0$

NOTE :- Order of operators are first compliment, followed by Product and Sum.

Examples (i) Find the values of $1\cdot 0+\overline{(0+1)}$

Solution $1\cdot 0+\overline{(0+1)}$
$\Rightarrow 0+\bar{1}$
$\Rightarrow 0+0$
$\Rightarrow 0$

(ii) $[(1+1)+(\bar{1}+0)]\cdot[(0\cdot\bar{0})+(1\cdot\bar{1})]$

Sol $[1+(0+0)]\cdot[(0\cdot 1)+(1\cdot 0)]$
$\Rightarrow (1+0)\cdot(0+0)$
$\Rightarrow 1\cdot 0$
$\Rightarrow 0$

2) Translate $1\cdot 0+\overline{(0+1)}=0$ into logical equivalence.

Sol $(T\wedge F)\vee\neg(F\vee T)=F$

3) Translate $(T\wedge T)\vee\neg F\equiv T$ into Boolean algebra identity

Sol $(1\cdot 1)+\bar{0}=1$

4) In Boolean algebra. $1+B=B$. False.

Reason $1+1=1$
$1+0=1$

Boolean operators	Logic
Sum $+$	$\vee$
Product $\cdot$	$\wedge$
Compliment $-$	$\neg$

5) The values of p, q, r, s which makes $\bar{p} + q + \bar{r} + s$ equal to zero.

Sol: $p = 1,\ q = 0,\ r = 1,\ s = 0$.

6) Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$

Sol

x	y	z	xy	$\bar{z}$	$xy + \bar{z}$
1	1	1	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	0	0	0	1	1
0	1	1	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	1	1

7) Find the values of the Boolean function represented by $F(x, y, z) = \bar{x}yz + x\bar{y}z + xy\bar{z}$

x	y	z	$\bar{x}$	$\bar{y}$	$\bar{z}$	$\bar{x}yz$	$x\bar{y}z$	$xy\bar{z}$	$\bar{x}yz + x\bar{y}z + xy\bar{z}$
1	1	1	0	0	0	0	0	0	0
1	1	0	0	0	1	0	0	1	1
1	0	1	0	1	0	0	1	0	1
1	0	0	0	1	1	0	0	0	0
0	1	1	1	0	0	1	0	0	1
0	1	0	1	0	1	0	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	0	1	1	1	0	0	0	0

Boolean Identities

1) $\bar{\bar{x}} = x$

2) $x + x = x$, $x \cdot x = x$ } Idempotent Laws. Eg: $1+1=1$, $0+0=0$, $1\cdot1=1$, $0\cdot0=0$

3) $x + 0 = x$, $x \cdot 1 = x$ } Eg: $1+0=1$, $0+0=0$, $1\cdot1=1$, $0\cdot1=0$ $\longrightarrow$ Identity laws.

4) $x+1=1$, $x \cdot 0 = 0$ } Eg: $1+1=1$, $0+1=1$, $1\cdot0=0$, $0\cdot0=0$ $\longrightarrow$ Domination laws.

5) Commutative laws: $x+y = y+x$, $x \cdot y = y \cdot x$.

6) Associative laws: $x + (y+z) = (x+y)+z$, $x(yz) = (xy)z$

7) Distributive Laws: $x + yz = (x+y)(x+z)$, $x(y+z) = xy + xz$.

8) De Morgan's laws: $\overline{xy} = \bar{x} + \bar{y}$, $\overline{x+y} = \bar{x}\,\bar{y}$

9) Absorption laws: $x + xy = x$, $x(x+y) = x$.

10) $x\bar{x} = 0$ (Eg: $1\cdot\bar{1} = 1\cdot0 = 0$, $0\cdot\bar{0} = 0\cdot1 = 0$)

11) $x + \bar{x} = 1$ (Eg: $1+\bar{1} = 1+0 = 1$, $0+\bar{0} = 0+1 = 1$)

Verifying Boolean Identities

For example verify the Distributive law $x+yz=(x+y)(x+z)$

x	y	z	$x+y$	$x+z$	yz	$x+yz$	$(x+y)(x+z)$
1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	0	1	1	1	0	1	1
1	0	0	1	1	0	1	1
0	1	1	1	1	1	1	1
0	1	0	1	0	0	0	0
0	0	1	0	1	0	0	0
0	0	0	0	0	0	0	0

As the last columns of the table agree, the identity agrees

Example Translate the distributive law $x+yz=(x+y)(x+z)$ into logical equivalence

Sol $x+yz=(x+y)(x+z)$

$\Rightarrow p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r)$

DUALITY :- The dual of a Boolean expression is obtained by interchanging sums and products and 0's and 1's

Examples 1) Find the dual of $x(y+0)$

Sol $x+(y\cdot 1)$

2) Find the dual of $\bar{x}\cdot 1+(\bar{y}+z)$

Sol $(\bar{x}+0)(\bar{y}\cdot z)$

3) Dual of $(x+y)\cdot z$

Sol $(x\cdot y)+z$

4) Dual of $(\bar{x}+y)(\overline{x\cdot 0})+1\cdot(\bar{y}+x)$

Sol $(\bar{x}\cdot y)+(\overline{x+1})(0+(\bar{y}\cdot x))$

Sum of Products expansions

1) Writing Boolean function as Sum of Products from given table

x	y	z	F	G
1	1	1	0	0
1	1	0	0	1
1	0	1	1	0
1	0	0	0	0
0	1	1	0	0
0	1	0	0	1
0	0	1	0	0
0	0	0	0	0

Example

Write the Boolean functions F and G from the the give table.

Sol. $F(x,y,z) = x \cdot \bar{y} z$

$G(x,y,z) = xy\bar{z} + \bar{x}y\bar{z}$

2) Writing Sum of Products for the given Boolean function

Eg. Find Sum of Products for the function $f(x,y,z) = (x+y)\bar{z}$

Sol.

x	y	z	$x+y$	$\bar{z}$	$(x+y)\bar{z}$
1	1	1	1	0	0
1	1	0	1	1	1
1	0	1	1	0	0
1	0	0	1	1	1
0	1	1	1	0	0
0	1	0	1	1	1
0	0	1	0	0	0
0	0	0	0	1	0

Ans. $f(x,y,z) = (x+y)\bar{z}$

$= xy\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$

Example

Which one of the following is the sum of products form

A) $(p+q)\cdot(r+s)$ ✓B) $p.q + r.s$ c) $(p)q(rs)$ D) none

Example

Find the sum of products of $f(x,y,z) = x+y+z$

Sol

x	y	z	$x+y+z$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	0

$$f(x,y,z) = xyz$$

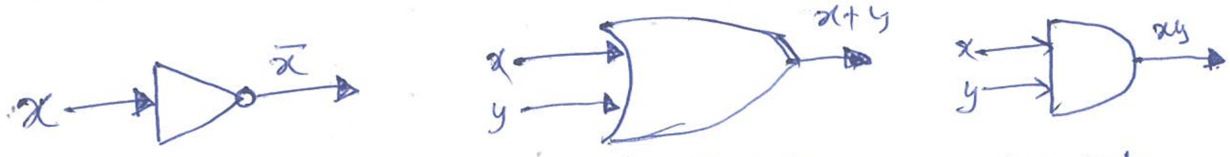
$$= xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}z$$

Example (Practice)

Find sum of products of

1) $f(x,y,z) = (x+z)\,y$
2) $f(x,y,z) = x$
3) $f(x,y,z) = x\bar{y}$

LOGIC GATES : Basic Types of Gates

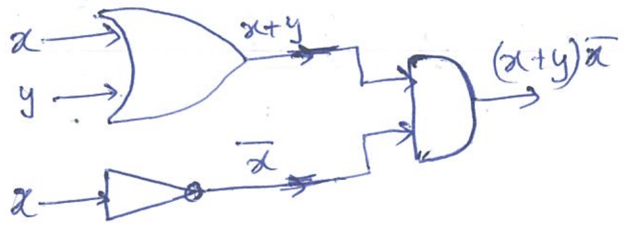


Inverter. OR gate. AND gate.

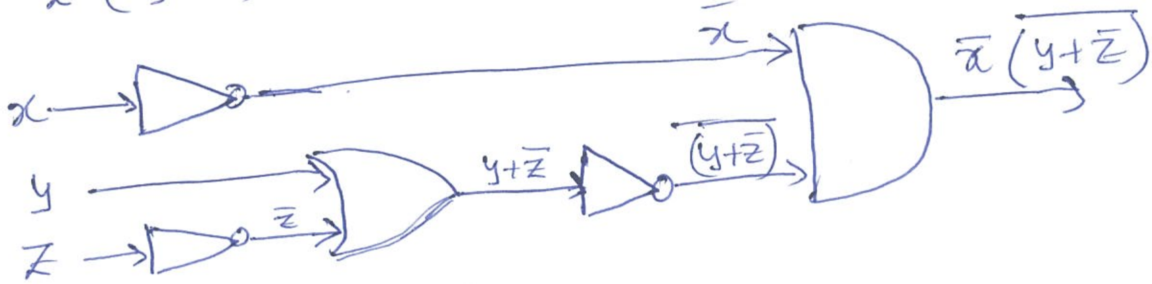
Combination of Gates – Construction

Examples Construct circuits that produce the following outputs.

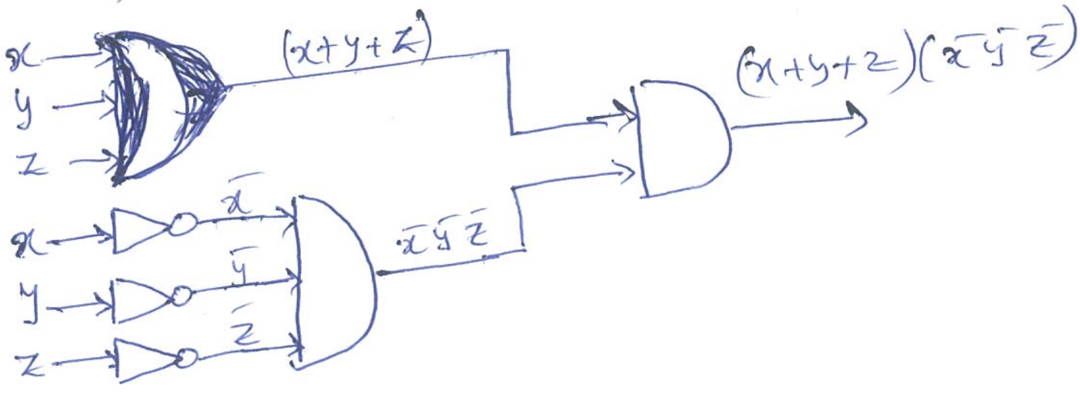
(i) $(x+y)\bar{x}$



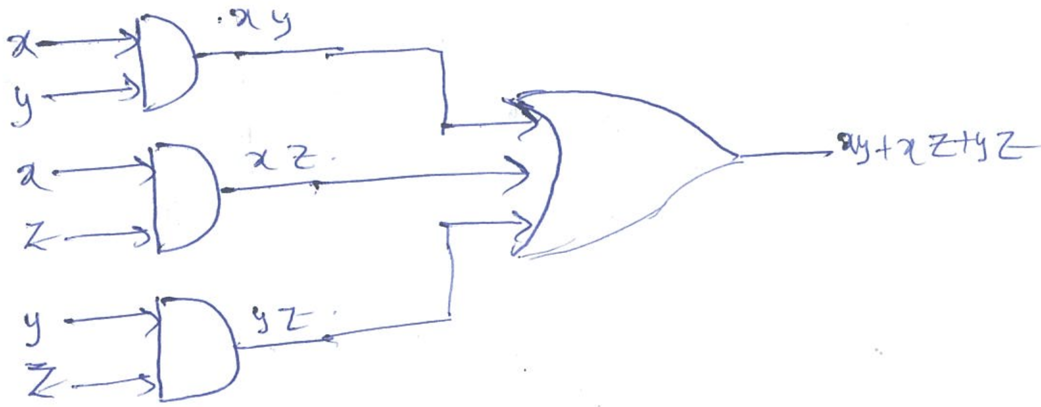
(ii) $\bar{x}\,\overline{(y+\bar{z})}$



(iii) $(x+y+z)(\bar{x}\bar{y}\bar{z})$



Problems Draw the circuit for $xy+xz+yz$.



Problem The resulting Boolean expression for the following circuit