# TREES

## CHAPTER-11

## WEEK-14

**TREE:-** A tree is a connected undirected graph with no simple circuit.

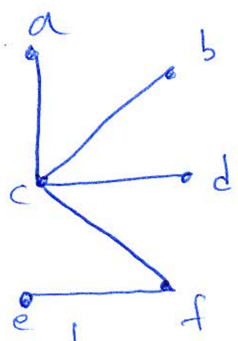
→ The meaning of connected means there must be a path between every pair of distinct vertices.

→ A tree cannot have multiple edges or loops.

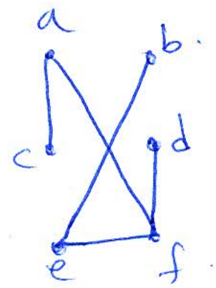
→ Any connected graph that contains no simple circuits is a tree

→ An undirected graph is a tree if and only if there is a unique simple path between any two of its vertices.

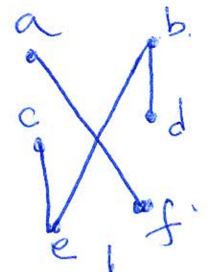
**Examples**



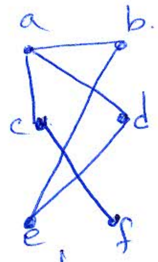

Tree.




Tree.




Not a tree
No paths between a, b ; a, c ; a, d ; a, e.
(Not connected)




Not a tree.
There is a circuit a, d, e, b, a

**ROOTED TREE:-** A tree in which one vertex has been designated as the root and every edge is directed away from the root is called a rooted tree.

* A tree together with its roots is a directed graph.
* The arrows indicating the directions of the edges can be omitted for a rooted tree, as the choice of root determines the direction of edges.

## TREE TERMINOLOGY:- (FOR ROOTED TREE).

**PARENT:-** The Parent of a vertex v is the unique vertex u such that there is a directed edge from u to v.

When u is the parent of v, v is child of u.

**SIBLINGS:-** Vertices with the same parent are called siblings.

**ANCESTORS:-** Ancestors of a vertex 'v' (other than root) are the vertices in the path from the root to this vertex 'v' including the root and excluding the vertex 'v'.

**DESCENDANTS:-** Descendants of a vertex 'v' are those vertices that have v as an ancestor.

**LEAF:-** A vertex is called a leaf if it has no children.

**INTERNAL VERTICES:** Vertices that have children are called internal vertices.

**SUBTREE:-** If a is a vertex of a tree, then a tree with a as root and its descendants and all edges incident to these descendants is called a subtree rooted at a

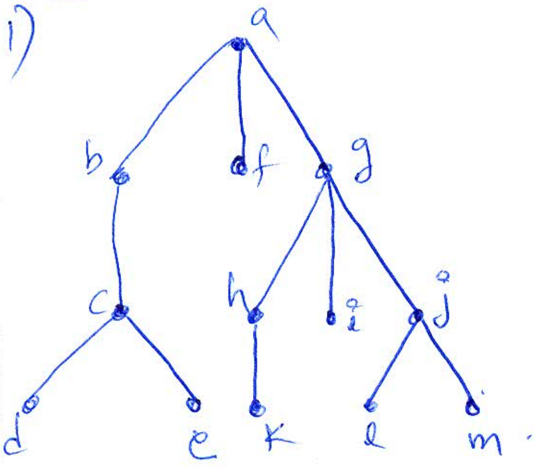
**LEVEL OF A VERTEX:-** The length of the unique path from the root to the vertex 'v' is called level of vertex v.

* Level of root is defined to be ZERO

**HEIGHT OF A ROOTED TREE:-** Length of the longest path from the root to any vertex (or) maximum of the levels of vertices, is called as height of the tree.

## Examples

1)



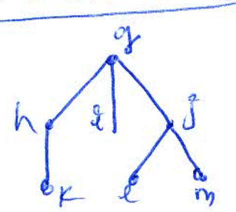

Level of root a = 0
height of tree = 3.
b, f, g — level 1
c, h, i, j - level 2.
d, e, k, l, m - level 3.

Root — a
Parent of c is b.
Parent of i is g
Parent of k is h.

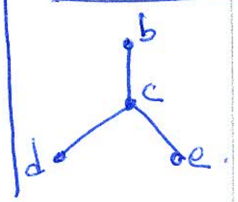
Descendents of b are c, d, e.
Descendants of g are h, i, j, k, l, m
Internal vertices are a, b, c, g, h, j

Children of g are h, i, j
Children of j are l, m.
Siblings of h are i, j
Siblings of l is j
Leaves are d, e, f, i, k, l, m

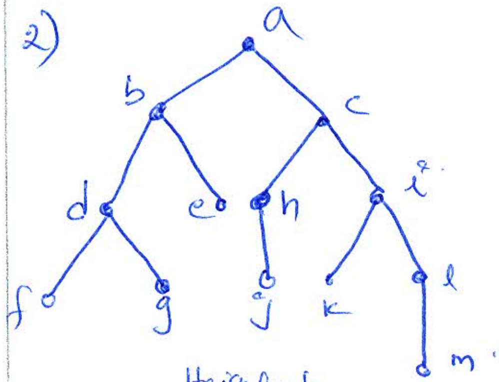
Subtree rooted at 'g'




Subtree rooted at b

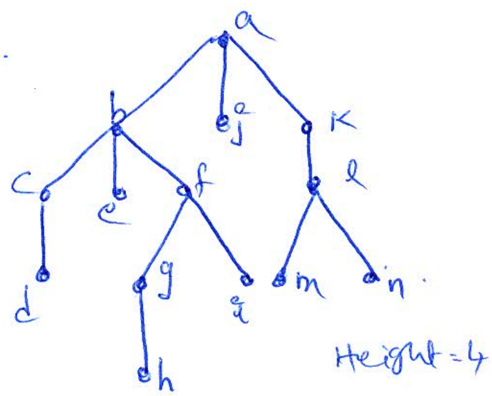



2)



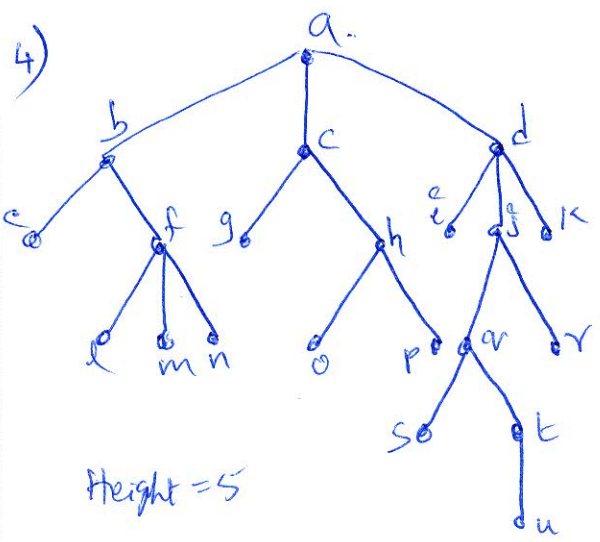

Height = 4

3).



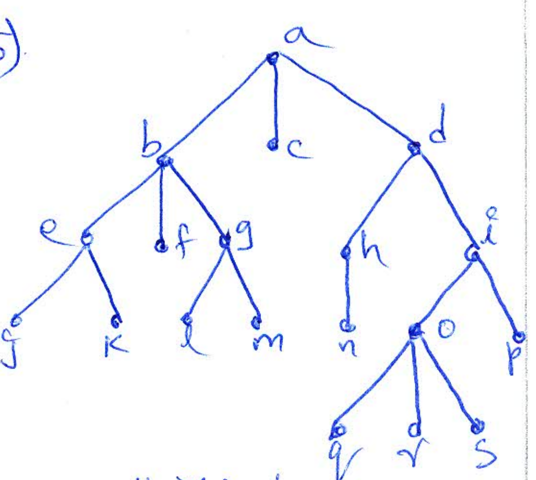

Height = 4

4)




Height = 5

5).



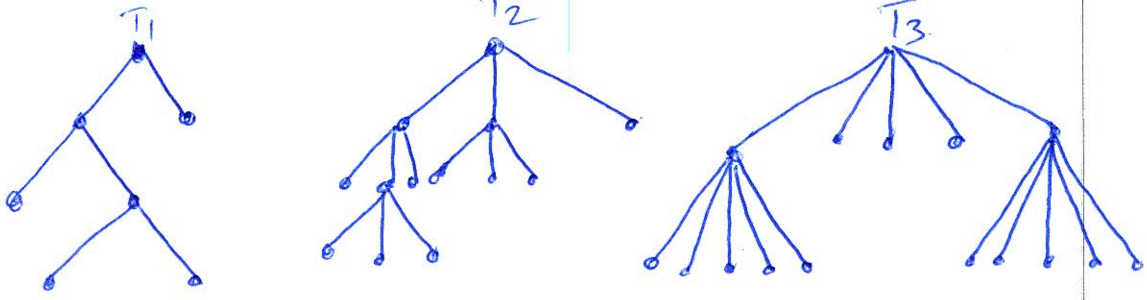

Height = 4

**M-ARY TREE:-** A rooted tree is called an m-ary tree if every internal vertex has not more than m children.

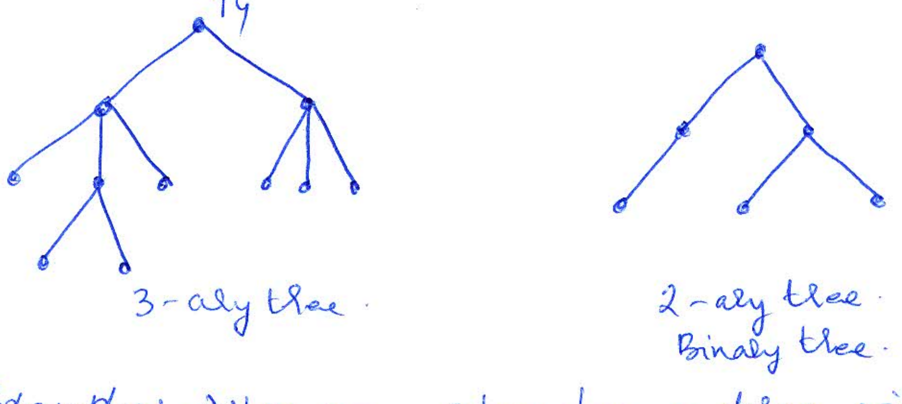
**FULL M-ARY TREE:-** A tree in which every internal vertex has exactly m-children is called as full m-ary tree.

**BINARY TREES:-** An m-ary tree with $m=2$ is called a binary tree

Examples:-



Full 2-ary tree (or) Full binary tree — Full 3-ary tree. — Full 5-ary tree



3-ary tree. — 2-ary tree. Binary tree.

Examples:- 1) How many edges does a tree with 10,000 vertices have?

Ans 9999

2) How many leaves does a full 3-ary tree with 100 vertices have?

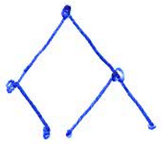
Sol $m=3$, $n=100$, $l = [(m-1)n+1]/m$

$$= \frac{(3-1)100+1}{3} = \frac{201}{3} = 67$$

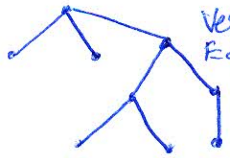
## PROPERTIES OF TREES:-

A tree with $n$ vertices has $(n-1)$ edges

For example



vertices = 6
Edges = 5



Vertices = 9
Edges = 8

## PROPERTIES OF FULL m-ARY TREE.

I)

| VERTICES | INTERNAL VERTICES | LEAVES |
|---|---|---|
| $n = mi+1$ | $i$ (GIVEN) | $l = (m-1)i+1$ |
| $n$ (GIVEN) | $i = \frac{n-1}{m}$ | $l = \frac{(m-1)n+1}{m}$ |
| $n = \frac{ml-1}{m-1}$ | $i = \frac{l-1}{m-1}$ | $l$ (GIVEN) |

Examples 1) For a 4-ary tree with 33 internal vertices find number of vertices and leaves.

Solution 4-ary tree $\Rightarrow m = 4$

Given $i = 33$.

vertices $n = mi+1$
$= 4(33)+1$
$= 133$

Leaves $l = (m-1)i+1$
$= (4-1)33+1$
$= (3)(33)+1$
$= 100$

2) For a 2-ary tree with 9 vertices, find internal vertices and leaves.

Solution 2-ary tree $\Rightarrow m = 2$

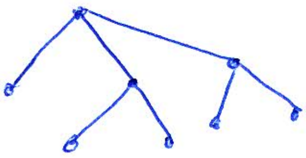
Given $n = 9$

$$i = \frac{n-1}{m} = \frac{9-1}{2} = \frac{8}{2} = 4$$

$$l = \frac{(m-1)n+1}{m} = \frac{(2-1)9+1}{2} = 5$$

II). In an m-ary tree of height h, the maximum number of leaves are $m^h$

(Eg 1)



This is a 3-ary tree $\Rightarrow m=3$.
height $h=2$.
Maximum number of leaves $= m^h = 3^2 = 9$

(E8) For a binary tree, of height 3, the maximum number of leaves?

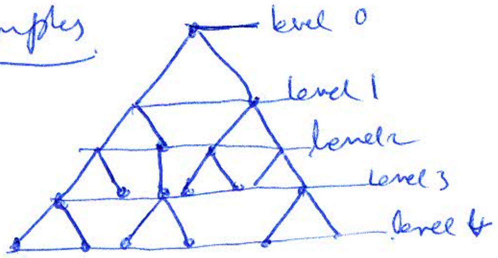
Sol. Binary tree $\Rightarrow m=2$
Given $h=3$.
Maximum number of leaves $= m^h = 2^3 = 8$

BALANCED M-ARY TREE :- A rooted m-ary tree of height h is balanced if all leaves are at levels h or (h-1).

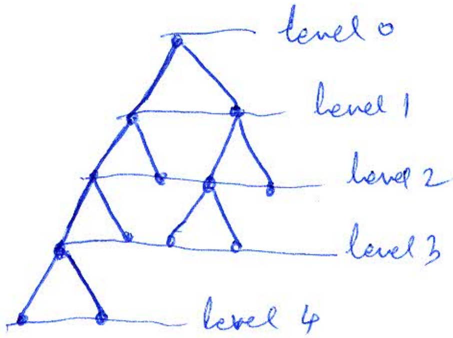
(OR).

Subtrees at each vertex contains paths of same length.

Examples




Balanced 2-ary tree
Height = 4
All leaves are at level 4 & 3




Not a Balanced tree.
Height = 4
All leaves are not at level 4 & 3.
Some leaves are at level 2.

Example :- How many edges does a full binary tree with 1000 internal vertices have?

Sol Full binary tree $\Rightarrow m=2$
Given $i = 1000$.
vertices $n = mi+1 = 2(1000)+1 = 2001$
Edges $= 2001 - 1 = 2000$